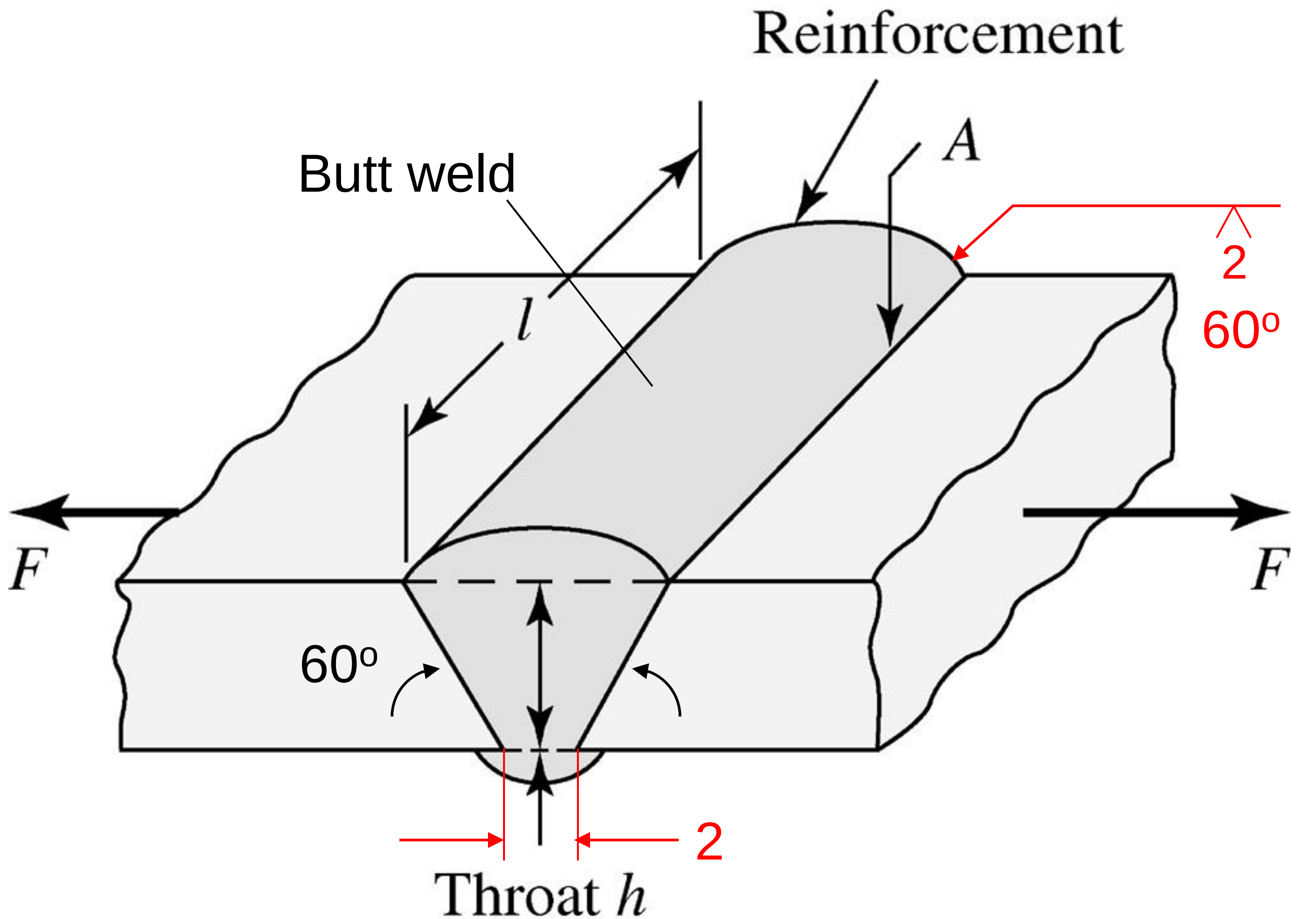
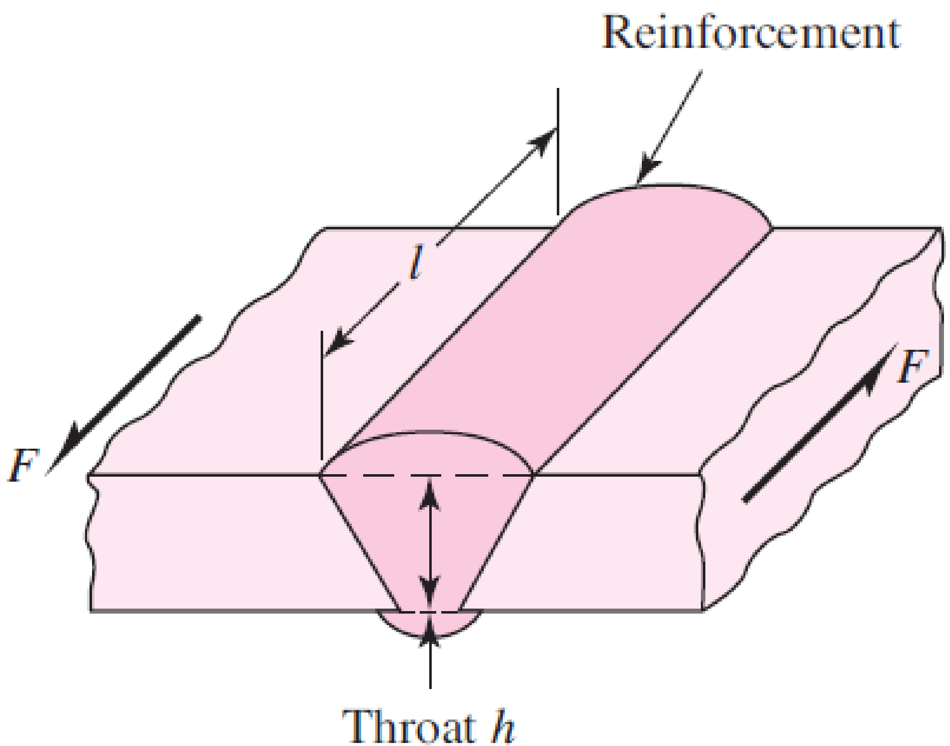
Welded Joints

Typical Butt Joint



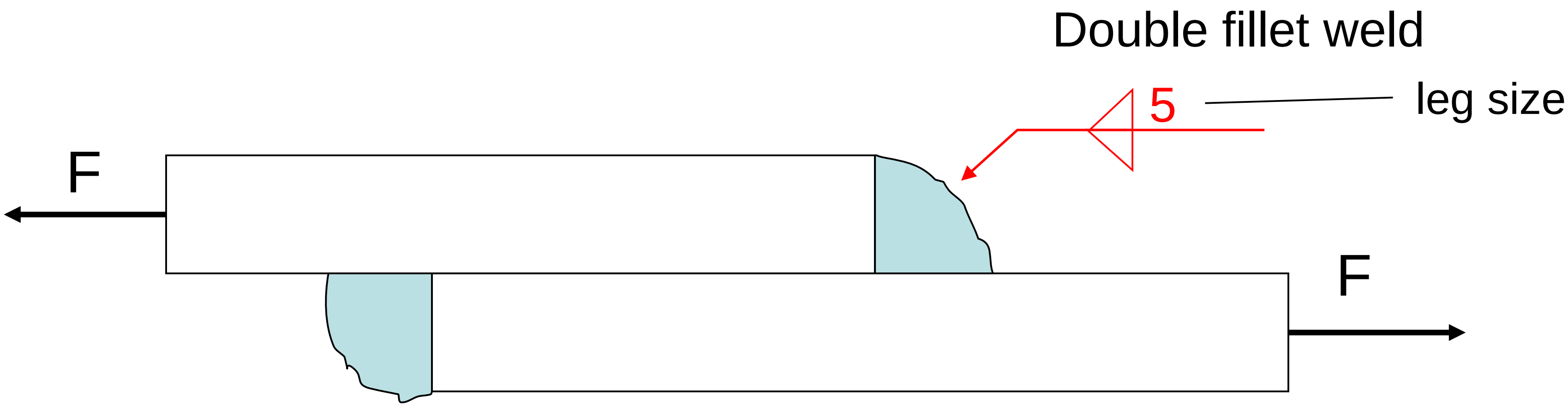
a) Tensile Loading

Typical Butt Joint

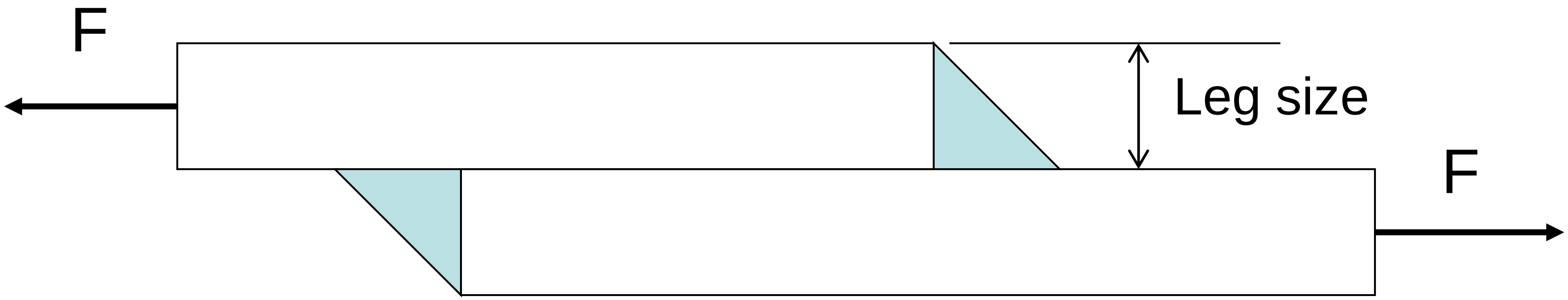


(b) Shear loading

Fillet Welds



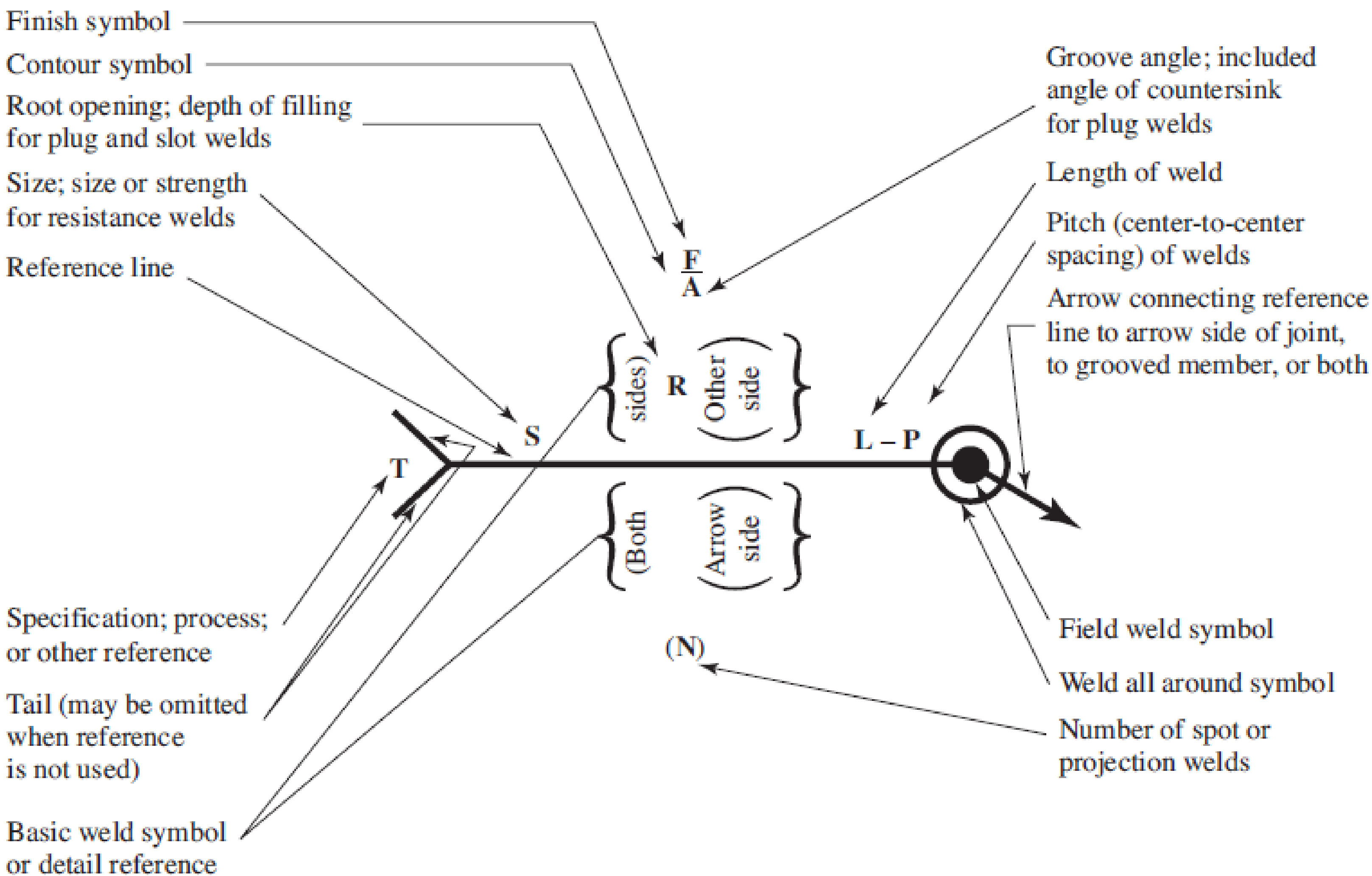
Lap Joint



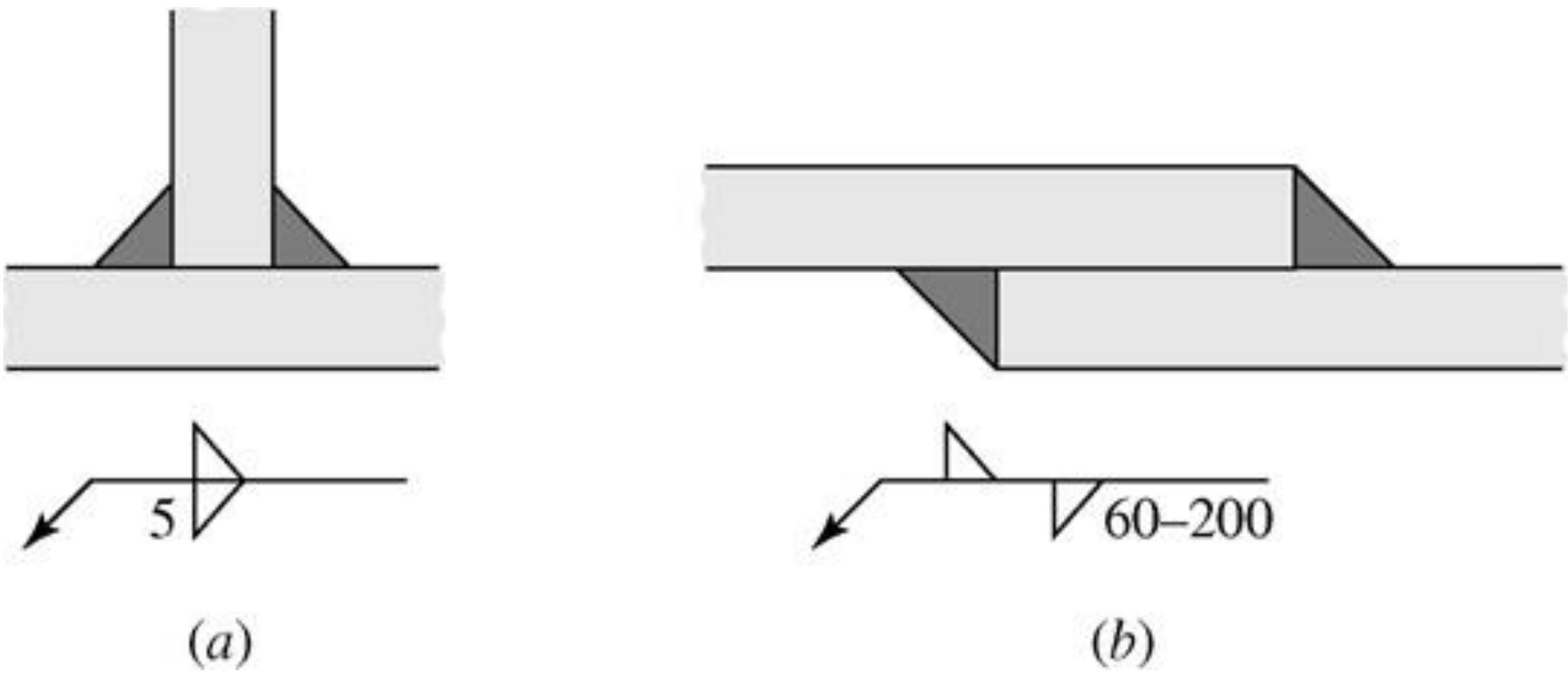
Idealized Fillet welds

Welds are considered to be 45 degree isosceles triangles

Welding Symbols (AWS)



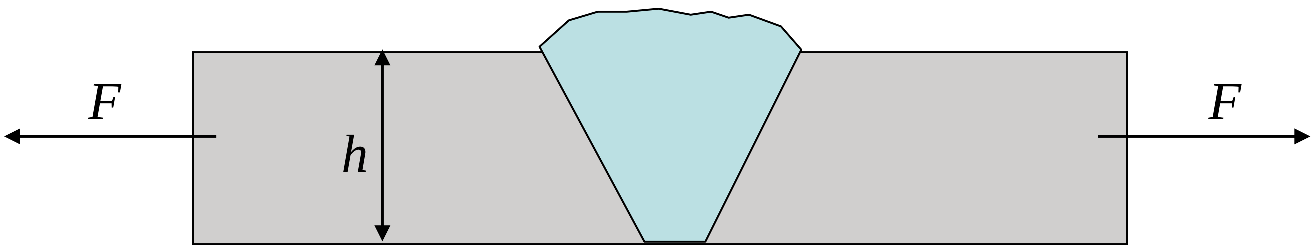
Welding Symbols



See the text book for other welding symbols and their use

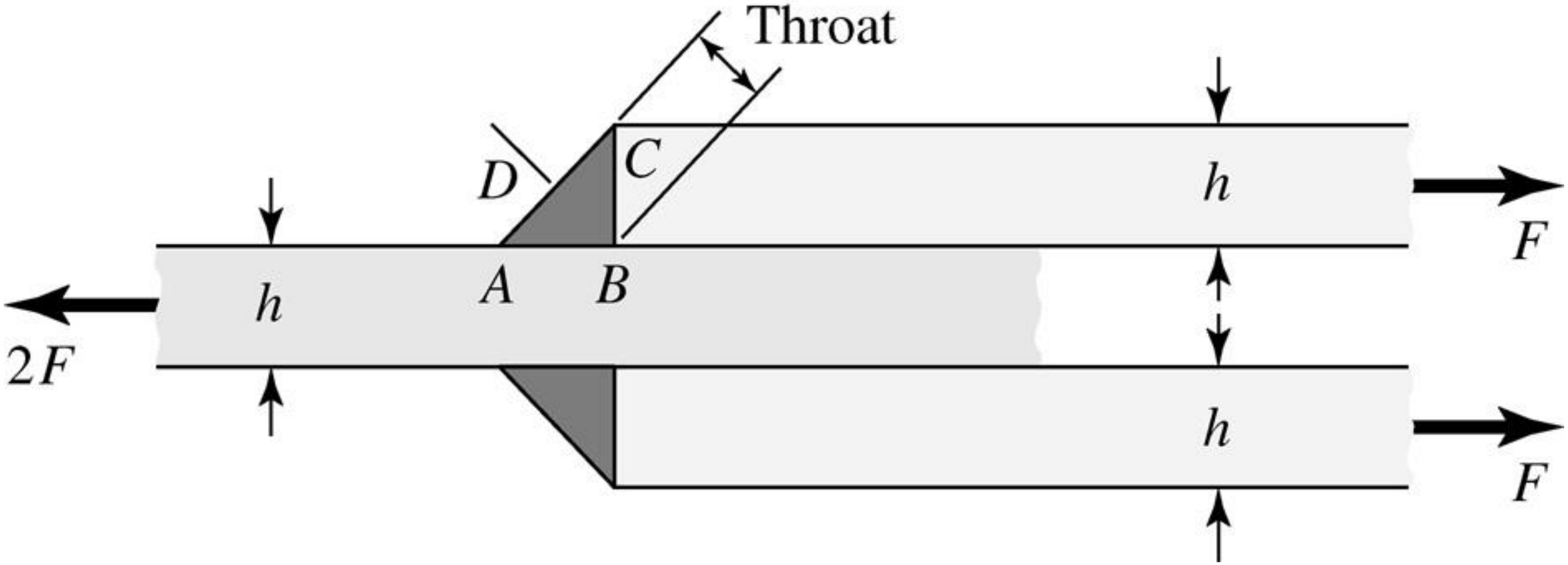
Analysis of Butt Welds

- Consider "centric loading" such that line of action of the applied force passes through the centroid of the weld line.



$$\sigma = \frac{F}{hl} \quad \text{(In case of shear loading } \tau = \frac{F}{hl} \text{)}$$

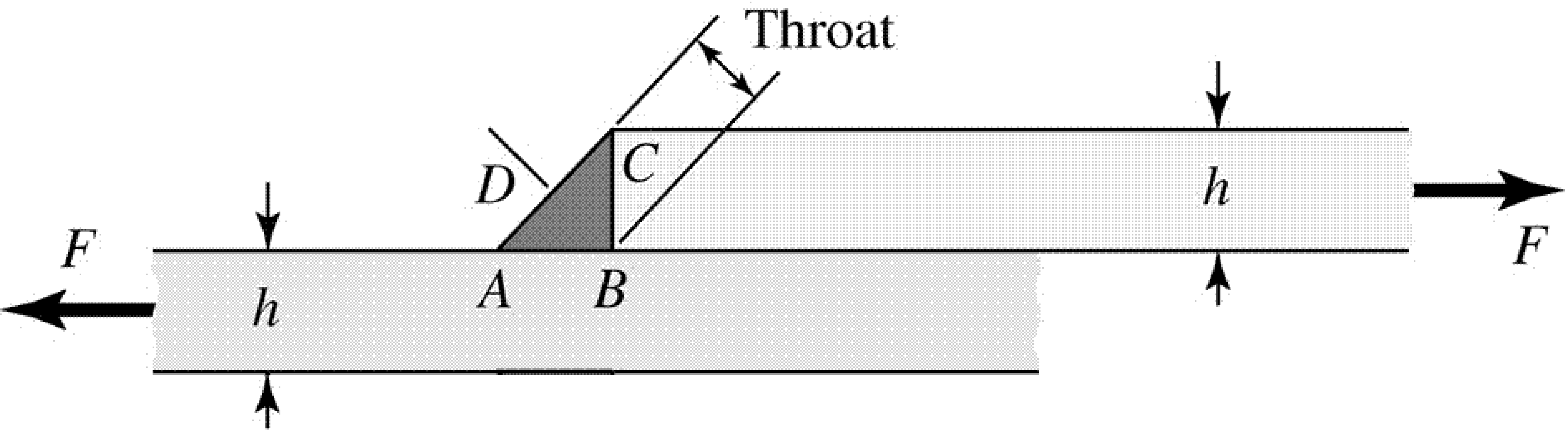
Analysis of Fillet Welds



Transverse Fillet Weld - Double Lap Joint

Design is based on throat area

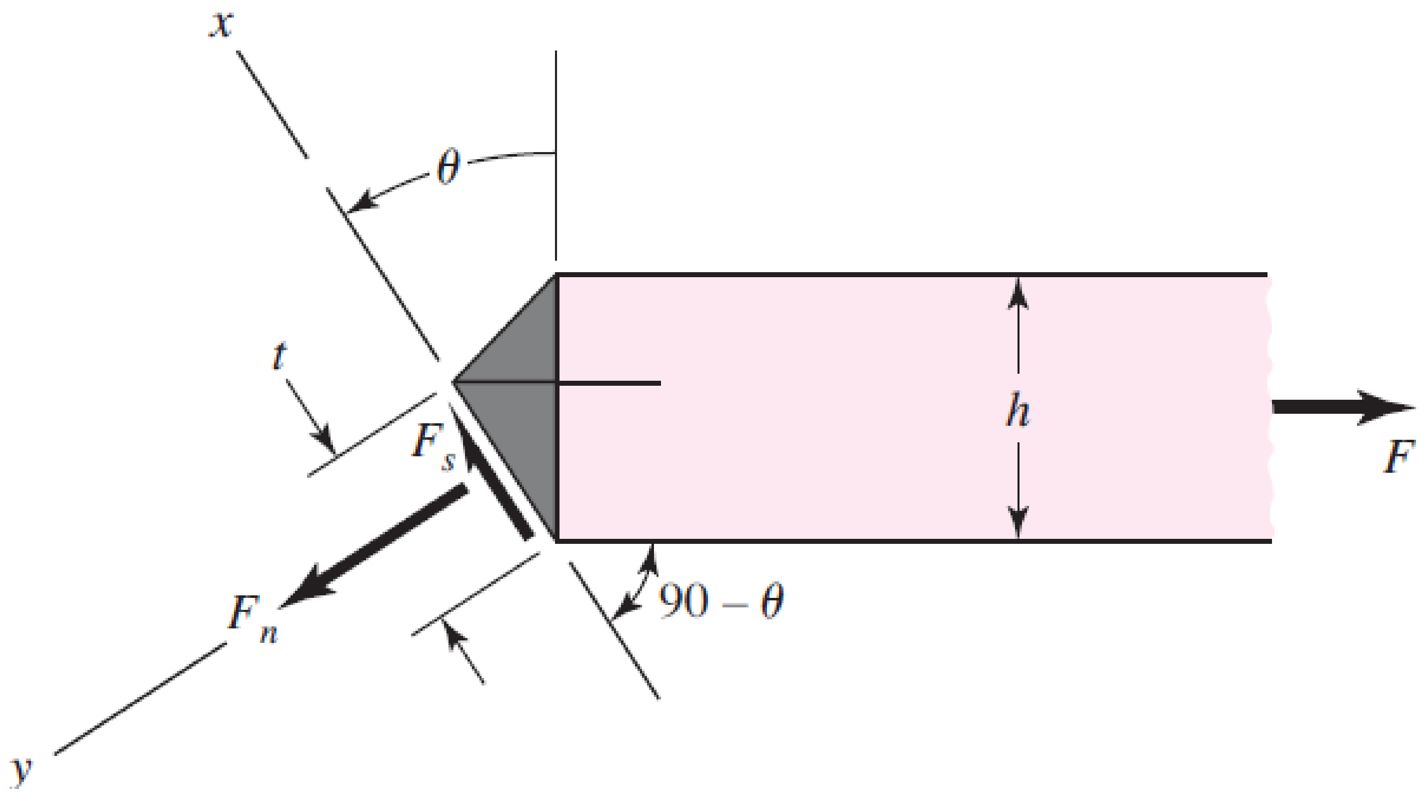
Single lap Joint



Bending effect due to offset between lines of actions of the forces will be neglected in the subsequent analysis

As it will become clear, design is based on the average shear stress acting on the throat area.

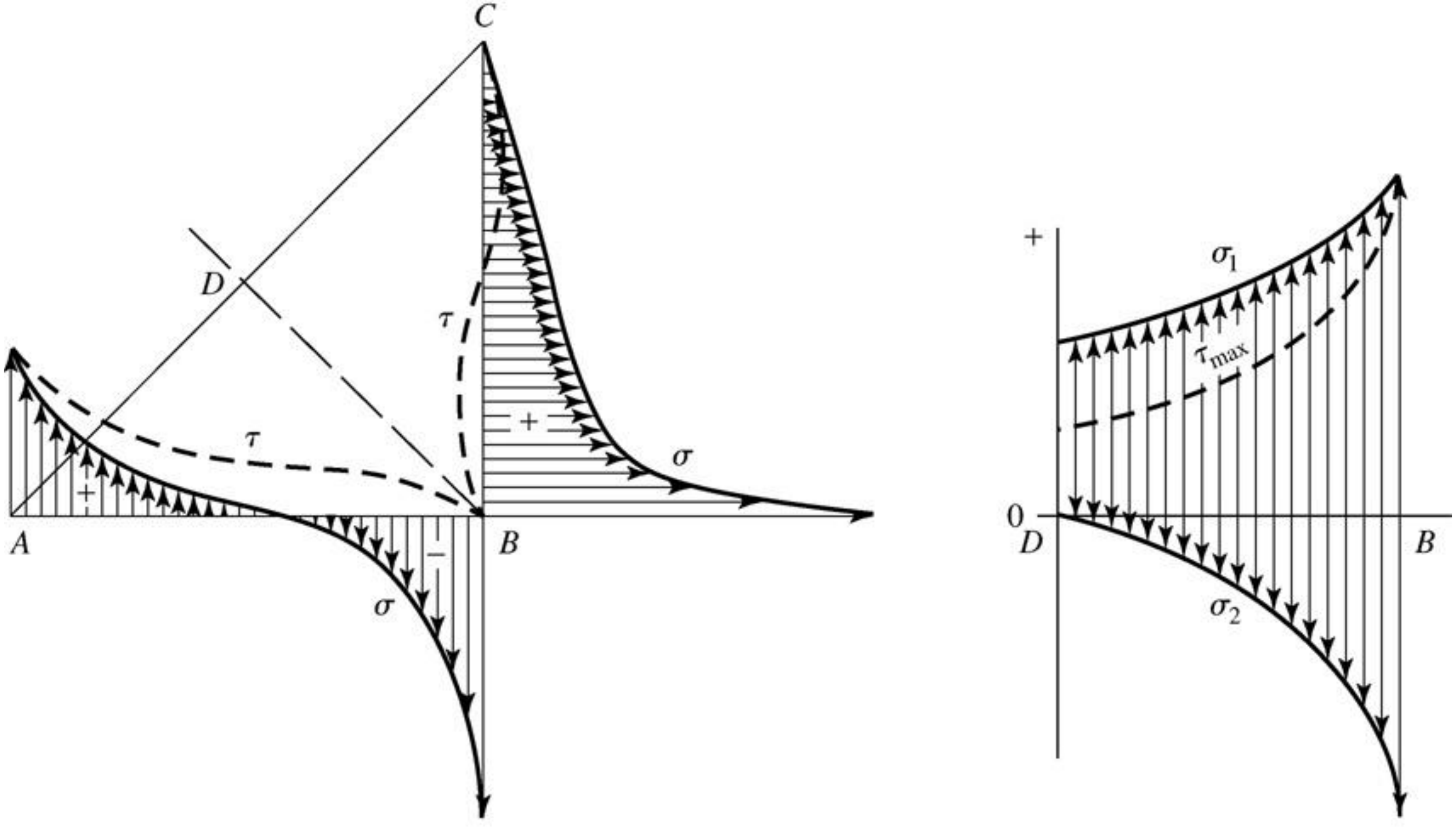
Free Body Diagram of Fillet Weld



Stresses in a fillet Weld

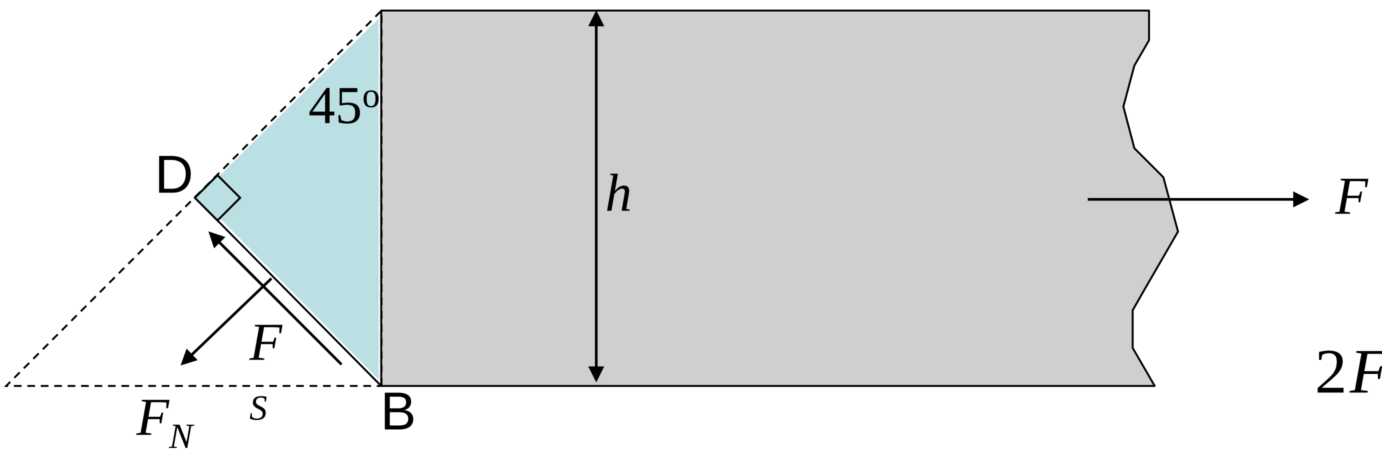
- From the free body diagram, it is evident that there are normal and shear stresses at a weld cross-section (given by θ in the previous slide).
- In general, these stresses are functions of θ and they are quite complicated because of the geometry and the residual stresses.
- Hence, these stress distributions can not be accurately determined by simple "strength of materials" type equations.

Residual Stresses in Welds



(a) (b)

Consider the average normal and shear stresses on the throat area :



$$\sum F_y = 0:$$

$$F_N = F_S$$

$$\sum F_x = 0:$$

$$2F_N \cos 45 = F$$

$$F_N = F_S = \frac{F}{2\cos 45}, \sigma = \tau = \frac{F/(2\cos 45)}{(h\cos 45)l}, \sigma = \tau = \frac{F}{2hl(\cos 45)^2} = \frac{F}{hl}$$

For the stress state given by σ and τ, the maximum shear stress can be found as follows:

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

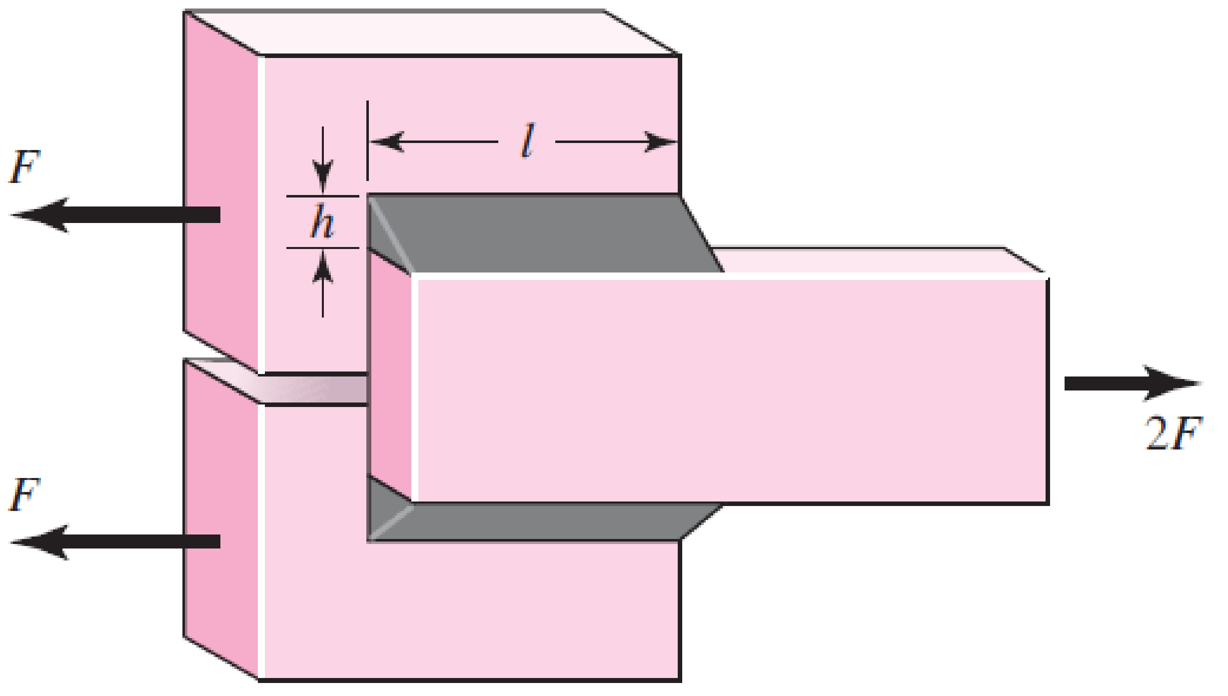
$$\tau_{\max} = \sqrt{\left(\frac{F}{2hl}\right)^2 + \left(\frac{F}{hl}\right)^2} = 1.118\frac{F}{hl}$$

Now note that $\tau_{\max}$ is actually smaller then the average shear stress based on the throat area and the applied force:

$$\tau = \frac{F}{(h\cos 45)l} \approx \frac{F}{0.707hl} \approx 1.414\frac{F}{hl}$$

- Then a simplified and conservative approach is adopted such that, the average shear stress acting on the weld throat area (which is indicated by DB in the previous slide) is used for design calculations.

Parallel Fillet Welds



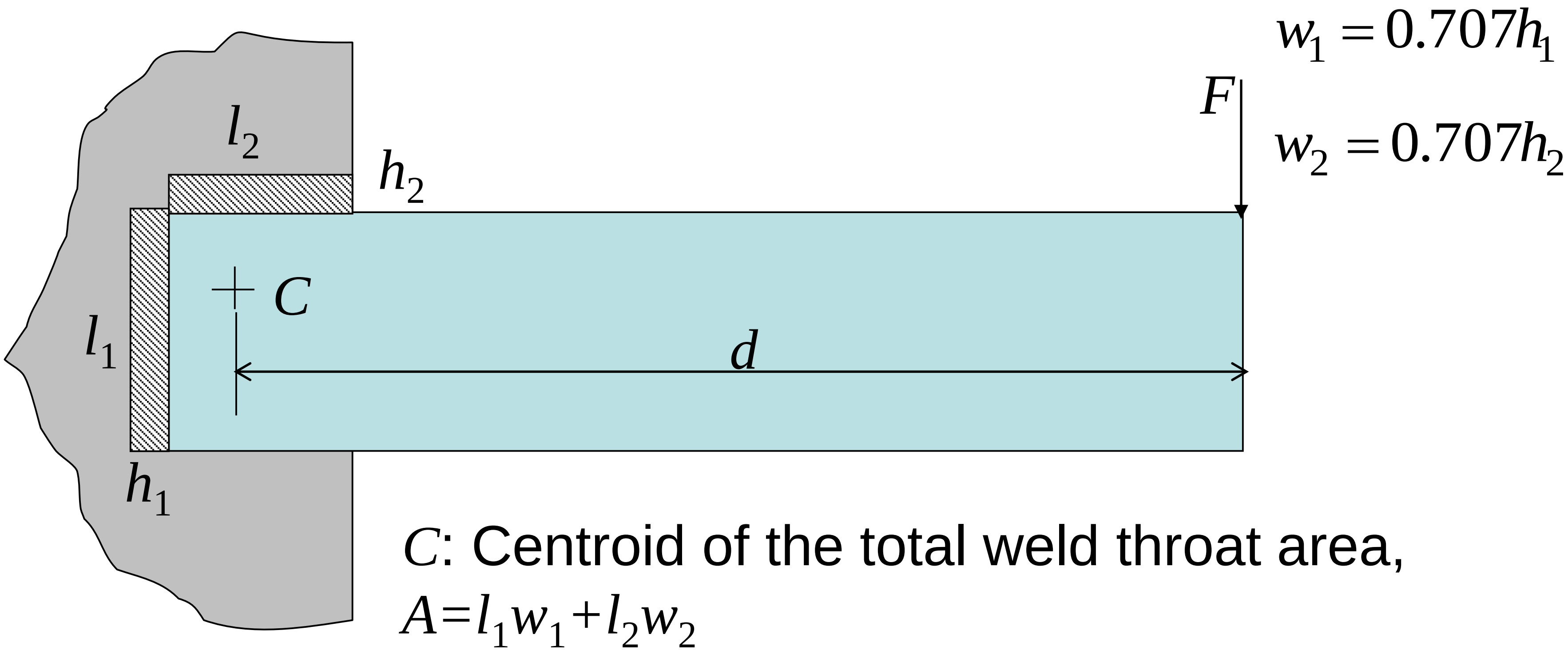
$$\tau = \frac{(F/2)}{(h\cos 45)l} \approx \frac{F}{1.414hl}$$

Eccentric loading of Welded Joints

- As in the case of riveted joints, if the line of action of resultant external load does <u>not</u> pass through the centroid of the weld area, the joint is said to be eccentrically loaded.
- Depending on the assumed form of deformation, eccentric loading can be divided into two types.
  - In-plane (torsion)
  - Out-of plane (bending)

In-Plane Eccentric Loading (Torsion)

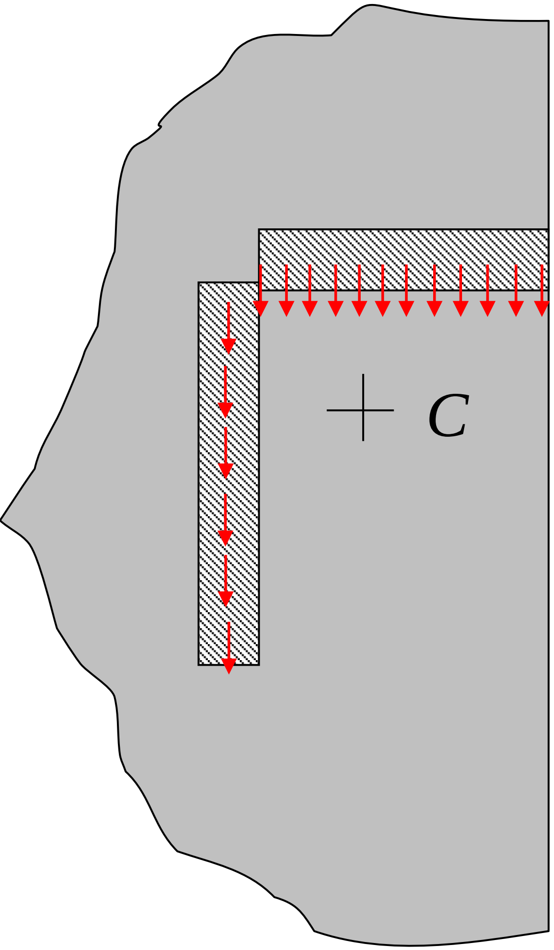
- The weld area (area consisting of weld lines) of the joint tends to rotate in its plane about its centroid.

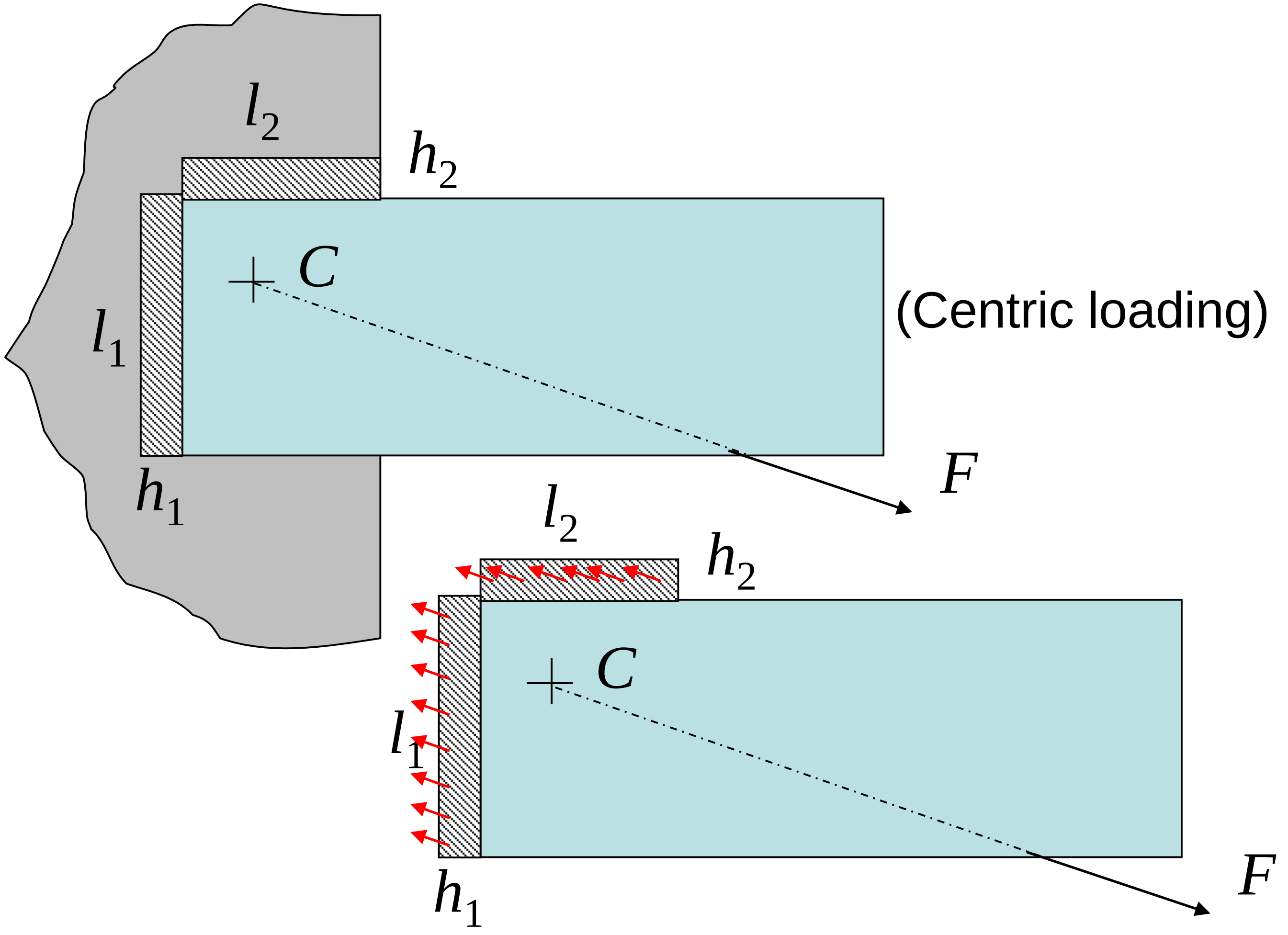


C: Centroid of the total weld throat area, $A = l_1 w_1 + l_2 w_2$

Primary shear stress : $\tau' = \dfrac{F}{A}$ is uniformly distributed over the weld area

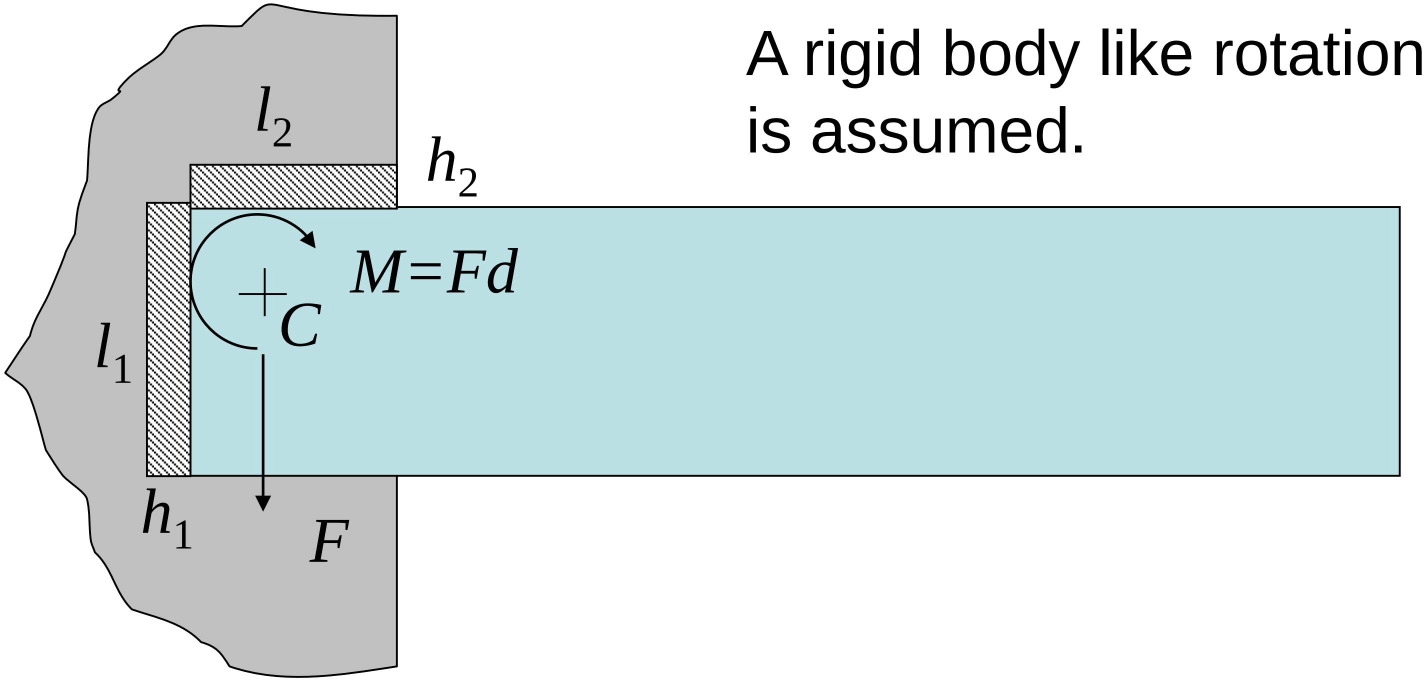
If the line of action of the load passes through the centroid, only primary shear exists.



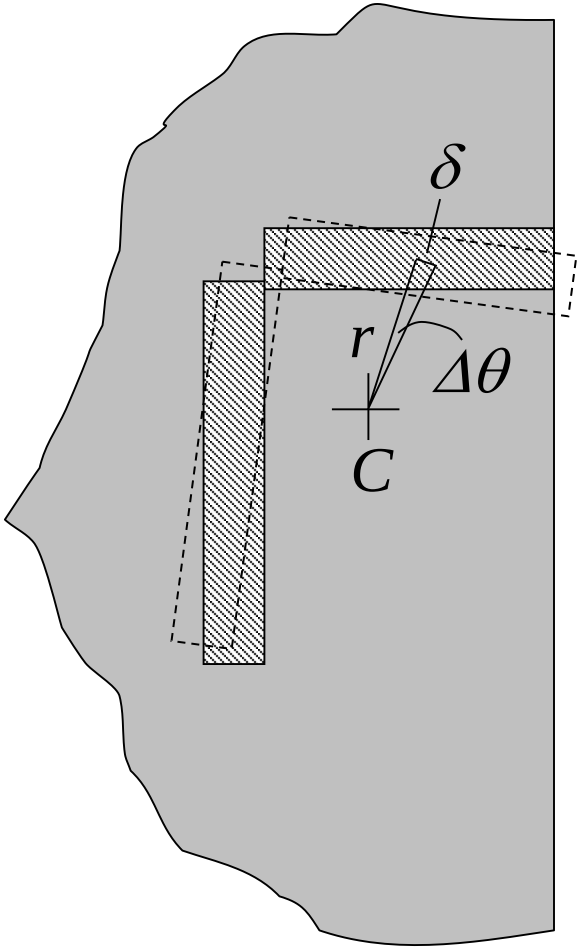


Secondary Shear Stress

- Note that the eccentric force creates a moment which tends to rotate the joint. This tendency is opposed by the secondary shear stresses



A rigid body like rotation about C is assumed.



$$\Delta\theta = \frac{\delta}{r} = c'$$

Angle of rotation for all the points on the weld area is constant. r is the distance between the centroid and an arbitrary point on the weld. δ is the displacement of that point.

It is further assumed that secondary shear stress at a point is directly proportional to displacement of that point.

$$\frac{\tau''}{r} \propto \frac{\delta}{r} = c'$$,therefore $\tau'' = cr$

Then the moment (torque) due to secondary shear can be expressed as

$$M = \int_A r\tau'' dA = \int_A cr^2 dA = c\int_A r^2 dA = cJ$$

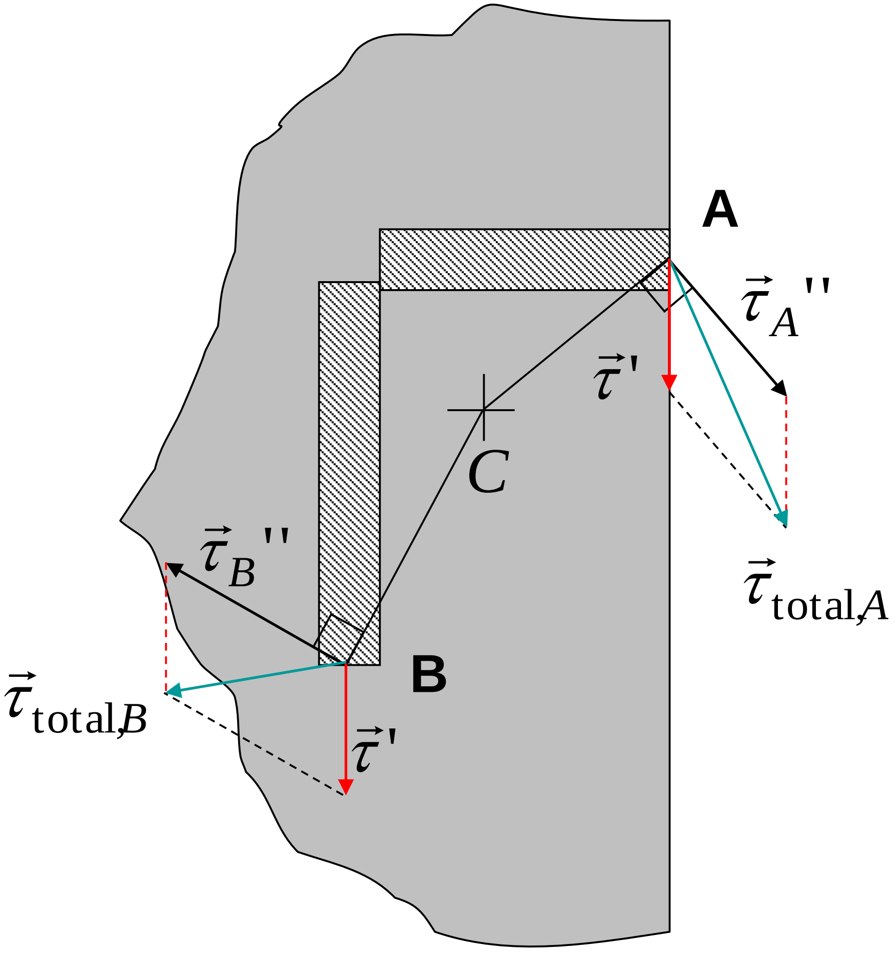
Hence, $M = cJ$ or $c = \dfrac{M}{J}$ leading to $\tau'' = \dfrac{Mr}{J}$

Note the similarity between the formula above and the torsion formula.
Here, J is the polar moment of weld throat area w.r.t. the centroid.
To combine primary and secondary shear stresses, we take their vector sum.

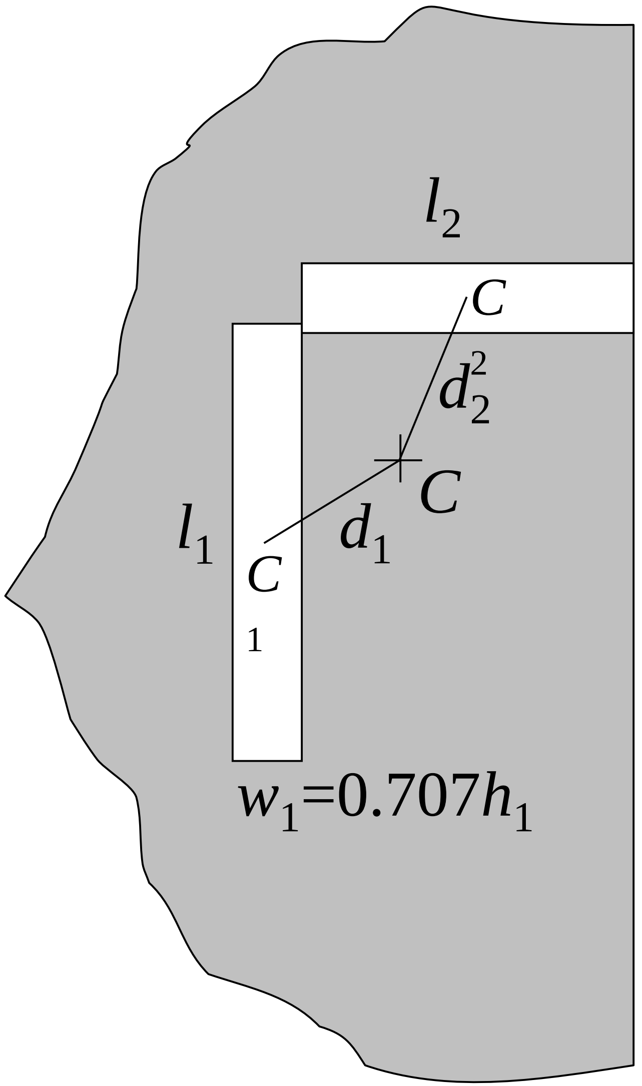
$$\vec{\tau}_{\mathrm{total}} = \vec{\tau}' + \vec{\tau}''$$

Note that the point(s) most distant from C will have the largest secondary shear stress(es). The point(s) which have the maximum total shear stress will become the critical points.

Superposition of primary and Secondary Shear



Calculation of J



$w_2=0.707h_2$

$w_1=0.707h_1$

let $w_1=w_2=w$

$$J_1 = \underbrace{\frac{1}{12} l_1 w^3}_{} + \frac{1}{12} l_1{}^3 w \approx \frac{1}{12} l_1{}^3 w$$

neglicible compared to 2nd term since $w<<l_1$.

$$J_2 = \underbrace{\frac{1}{12} l_2 w^3}_{} + \frac{1}{12} l_2{}^3 w \approx \frac{1}{12} l_2{}^3 w$$

neglicible compared to 2nd term since $w<<l_2$.

$$J = \frac{1}{12} l_1{}^3 w + d_1{}^2 w l_1 + \frac{1}{12} l_2{}^3 w + d_2{}^2 w l_2$$

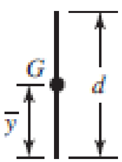
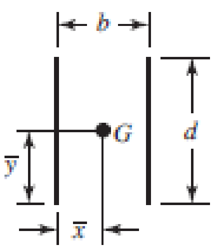
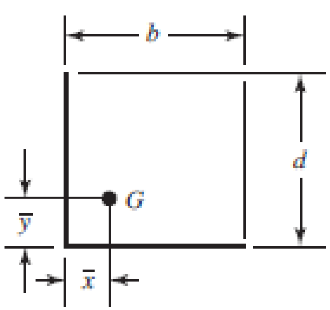
$$J = w\left(\frac{1}{12} l_1{}^3 + d_1{}^2 l_1 + \frac{1}{12} l_2{}^3 + d_2{}^2 l_2 \right)$$

Torsional Properties of Fillet Welds

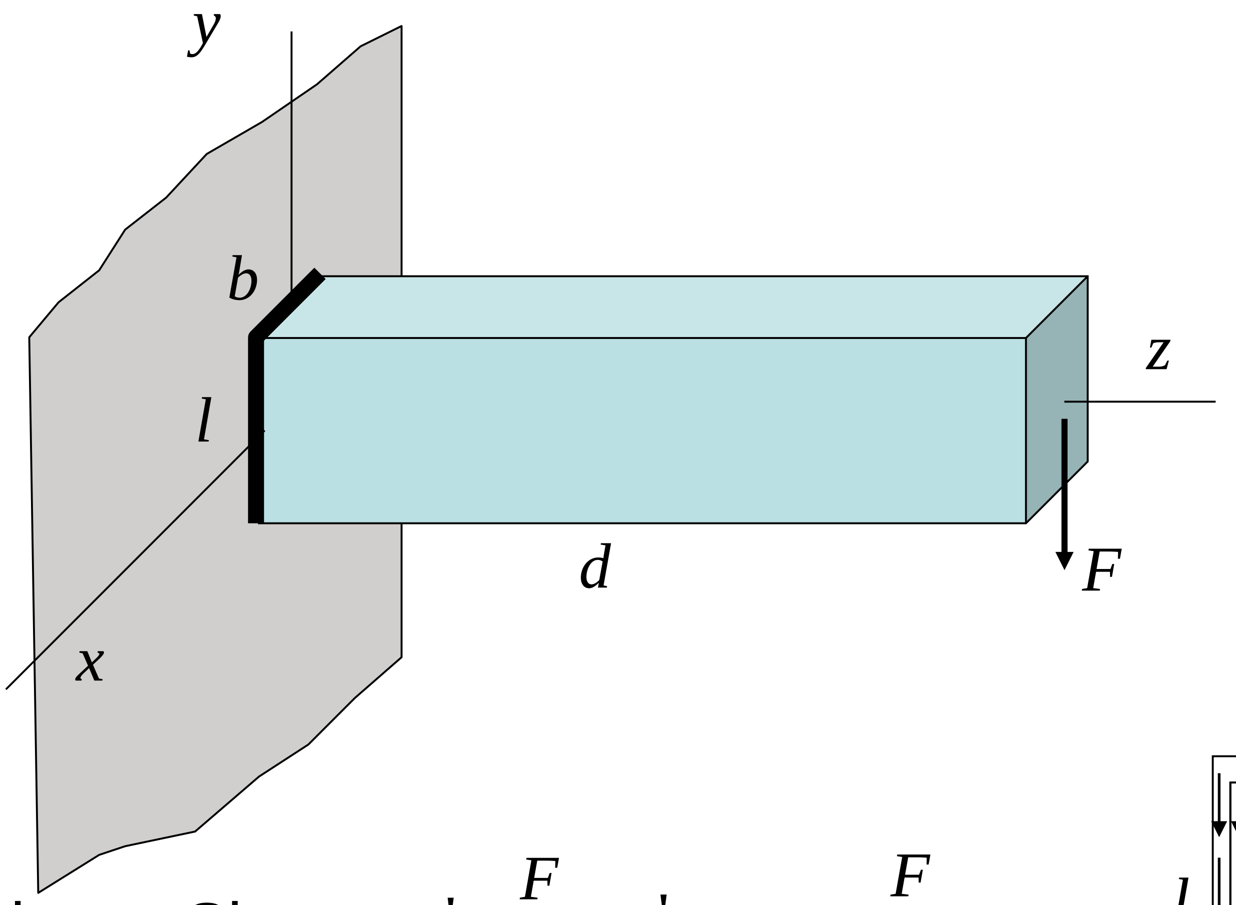
$$J = \sum_{i=1}^{n} A_i \left(\frac{l_i^{\,2}}{12} + d_i^{\,2} \right)$$

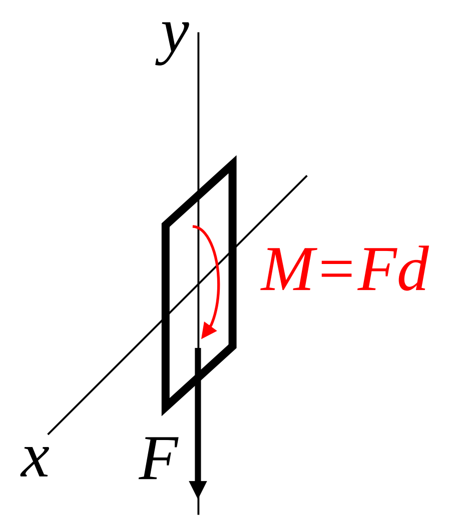
Table 9-1

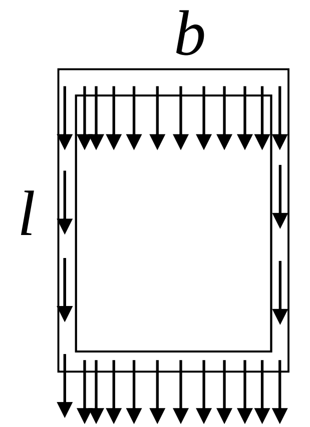
Torsional Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
	$A = 0.70\,hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
	$A = 1.41\,hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{d(3b^2 + d^2)}{6}$
	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2(b+d)}$ $\bar{y} = \dfrac{d^2}{2(b+d)}$	$J_u = \dfrac{(b+d)^4 - 6b^2d^2}{12(b+d)}$

Out-of-Plane Eccentric Loading (Bending)





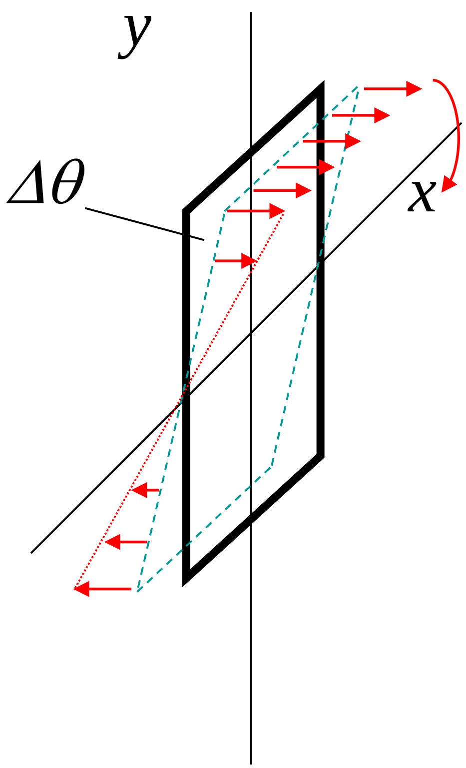


Primary Shear: $\tau' = \dfrac{F}{A}$ $\quad \tau' = \dfrac{F}{2(b+l)0.707h}$

uniformly distributed over the area

SECONDARY SHEAR:

Again assume rigid-body like rotation of welds with constany angle of rotation for all the points on the weld area. Rotation due to M takes place about x-axis therfore y is the distance between the axis of rotation and an arbitrary point on the weld. δ is the displacement of that point.



It is further assumed that secondary shear stress at a point is directly proportional to displacement of that point.

$$\delta = y\Delta\theta$$

$$\tau'' = c'\delta = y\underbrace{c'\Delta\theta}_{c} = cy$$

$$M = \int_A y\tau''dA = \int_A cy^2dA = c\int_A y^2dA = cI_{xx}$$

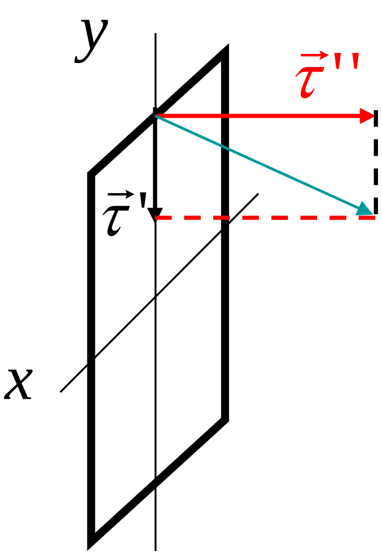
$c = \dfrac{M}{I_{xx}}$ leading to $\tau'' = \dfrac{My}{I_{xx}}$ maximum value will be at maximum y.

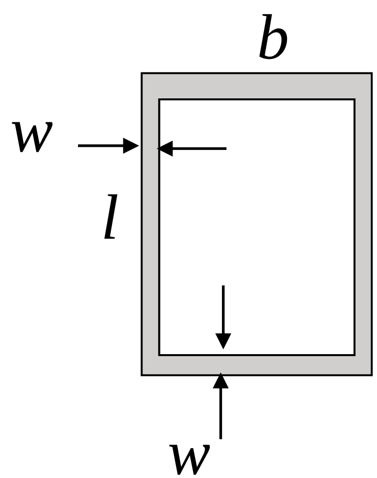
at $y=l/2$, $\tau''=\dfrac{Ml}{2I_{xx}}$

$\vec{\tau}_{\text{total}}=\vec{\tau}'+\vec{\tau}''$ but now prlimary and secondary shear stresses are perpendicular to each other.

$\vec{\tau}_{\text{total}}=\sqrt{\tau'^2+\tau''^2}$

I_{xx} is based on throat size.





$$w=0.707h$$

negligible

$$I_{xx}=2\left\{\cancel{\frac{1}{12}bw^3}+bw\left(\frac{l}{2}\right)^2+\frac{1}{12}l^3w\right\}$$

$$I_{xx}\approx 2\underbrace{\left\{b\frac{l^2}{4}+\frac{1}{12}l^3\right\}}_{(I_{xx})_u}w$$

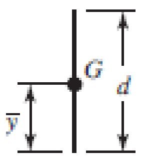
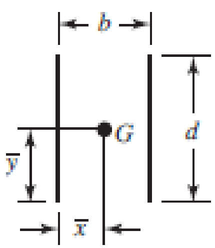
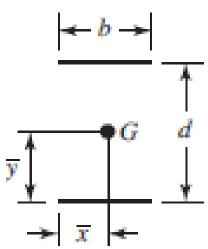
unit 2nd moment of area

Bending Properties of Fillet Welds

$$I_{xx} = 0.707h\left(I_{xx}\right)_u$$

Table 9–2

Bending Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Moment of Area
	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$

Design Equations

- Static Loading $$\tau = \frac{S_{sy}}{n}$$
- Fatigue Loading (Goodman) $$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$
- Fatigue Loading (Soderberg) $$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{sy}} = \frac{1}{n}$$

$S_{se} = 0.5S_e$ (MSST) $S_{su} = 0.67S_{ut}$ $S_{sy} = 0.5S_y$ (MSST)

$S_{se} = 0.577S_e$ (DET) $S_{sy} = 0.577S_y$ (DET)

$S_e = k_a k_b k_c k_d k_e S_e'$ $S_e' = 0.5S_{ut}$ (applicable to weld material)

$$k_e = \frac{1}{K_{fs}}$$

Table 9-5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Guidelines

- Use S_{ut}, S_y, S_e of weld (electrode) or parent material whichever is smaller.
- Also check critical points in the parent material when necessary.
- For k_a, "as forged"surface condition can be used.
- For k_b, if shear stress is uniform then $k_b=1$.
- If shear stress is not uniform, k_b can be found by using equivalent size (d_{eff}) concept.