Fasteners and Power Screws II

Tension Connections

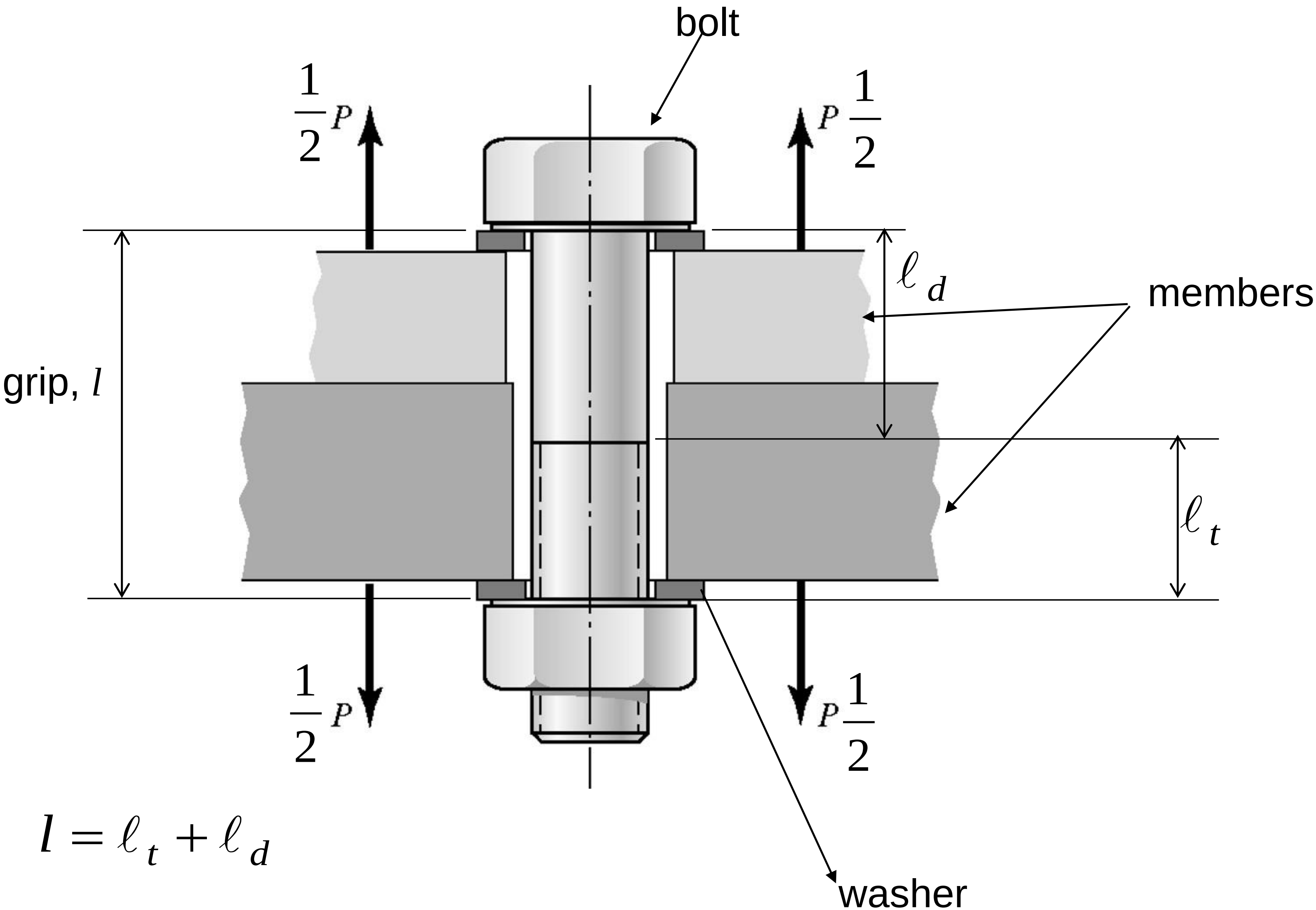
The first issue to be addressed is Joint Stiffness.

Twisting the nut stretches the bolt to produce clamping force.
Clamping force (initial tensile load) is also called pre-tension or preload.
While the bolt is stretched, members are compressed.
Both the bolt and members are assumed to be made of linear elastic materials, so they behave like linear springs.
Hence before going into the force analysis, spring constants of bolt and members must be determined.

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

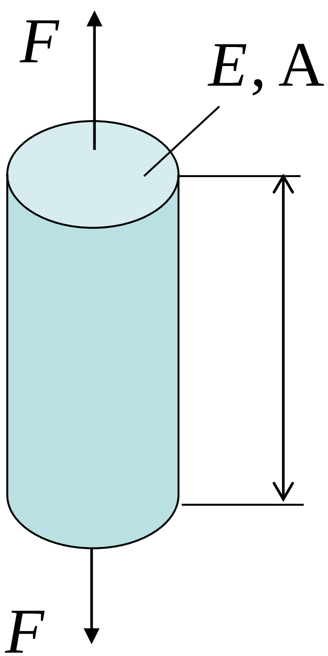
Nominal Major Diameter d mm	Coarse-Pitch Series Pitch p mm	Coarse-Pitch Series Tensile-Stress Area A_t mm²	Coarse-Pitch Series Minor-Diameter Area A_r mm²	Fine-Pitch Series Pitch p mm	Fine-Pitch Series Tensile-Stress Area A_t mm²	Fine-Pitch Series Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884


bolt
$\frac{1}{2}P$
$P\frac{1}{2}$
ℓ_d
members
grip, l
ℓ_t
$\frac{1}{2}P$
$P\frac{1}{2}$
washer

$$l = \ell_t + \ell_d$$

Spring constant for bolt

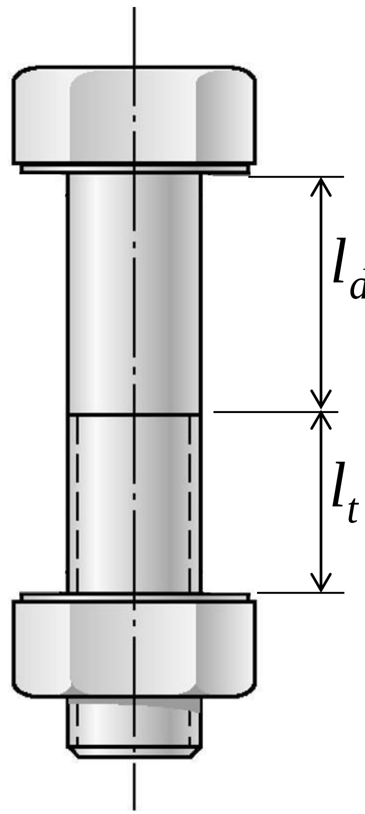
recall that for a bar, $\delta = \dfrac{Fl}{EA}$



$$k = \frac{F}{\delta}$$

$$k = \frac{EA}{l}$$

Therefore for a bolt



A_t : tensile stress area

A_d : Major diameter area

$$k_d = \frac{EA_d}{l_d}$$

$$k_t = \frac{EA_t}{l_t}$$

$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}$$

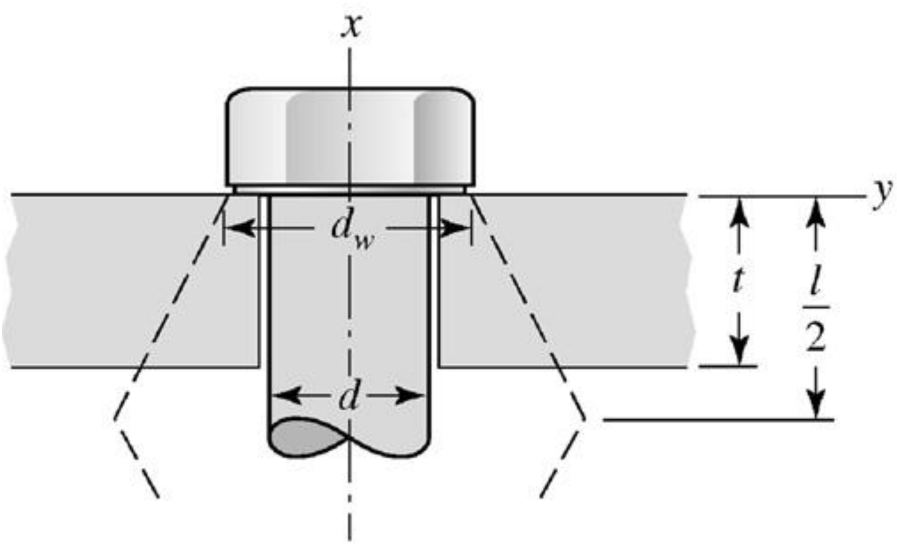
If the effect of thraded portion is neglected

$$k_b \approx A_d E / l$$

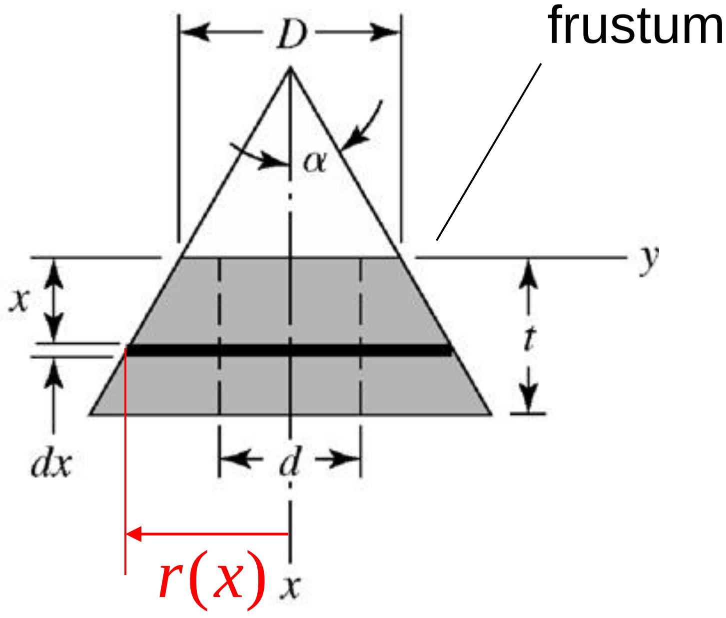
Spring constant for members

The material inside the conical frustum is assumed to be compressed.
A "pressure cone" is assumed to exist.

We take $\alpha=30^o$ unless otherwise is specified.



(a)



(b)

$$r(x)=\frac{D}{2}+x\tan\alpha$$

$$d\delta = \frac{F\,dx}{EA(x)} \qquad A(x) = \pi\left[r(x)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$A(x) = \pi\left[\left(x\tan\alpha + \frac{D+d}{2}\right)\left(x\tan\alpha + \frac{D-d}{2}\right)\right]$$

Total deformation of conical frustum is

$$\delta = \frac{F}{\pi E}\int_0^t \frac{dx}{\left(x\tan\alpha + \frac{D+d}{2}\right)\left(x\tan\alpha + \frac{D-d}{2}\right)}$$

integrating

$$\delta = \frac{F}{\pi E d\tan\alpha}\ln\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}$$

Spring rate(stiffness) of conical frustum is

$$k = \frac{F}{\delta} = \frac{\pi E d\tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}}$$

k should be calculated for each frustum in the joint. Then member stiffness becomes :

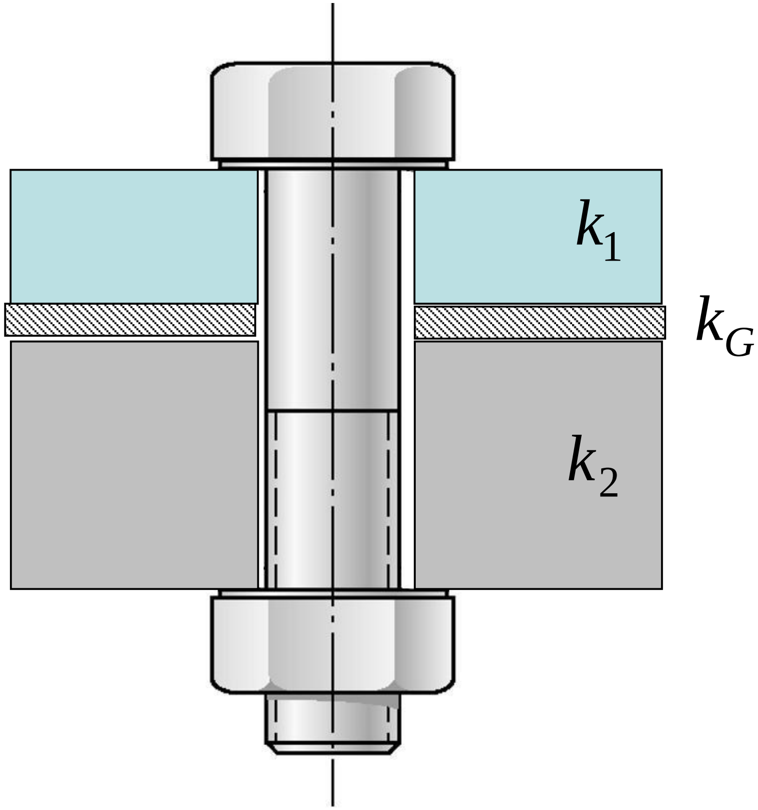
$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

Washer diameter d_w can be taken as : $d_w \approx 1.5d$

If the members of the joint have the same Young modulus E with symmetrical frusta back to back, they would act as two identical springs in series. Hence k_m simplifies to (l:grip, α=30°) ;

$$k_m = \frac{1.8138Ed}{2\ln\left(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)}$$

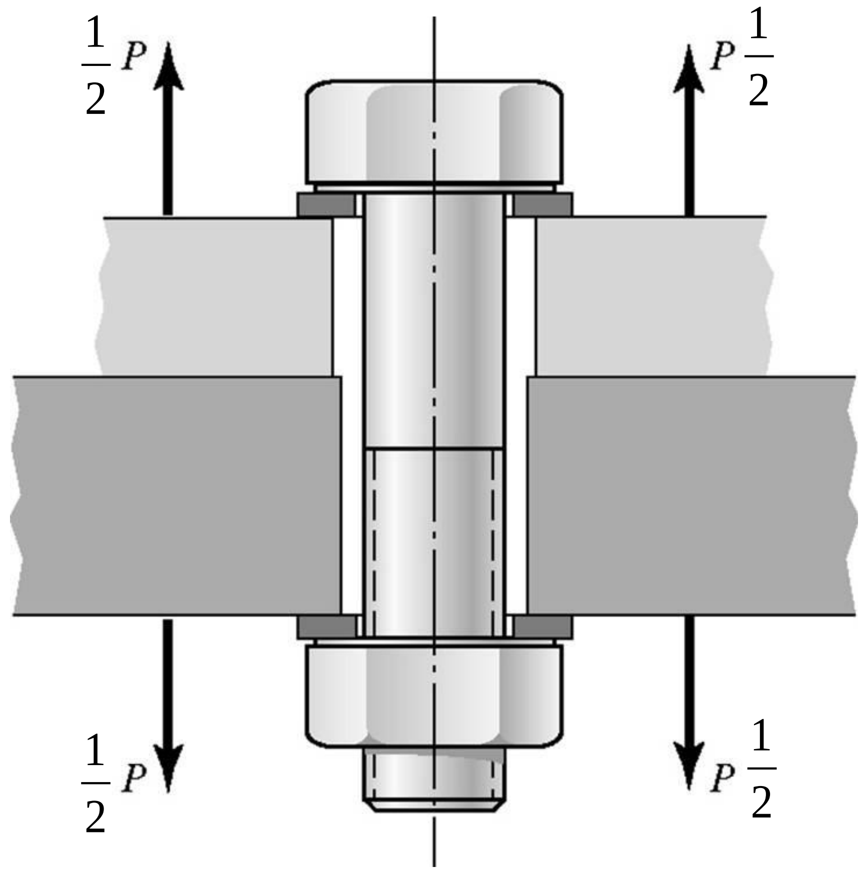
If there is a soft gasket in the joint such that $k_G << k_1, k_2$
then $k_m \approx k_G$



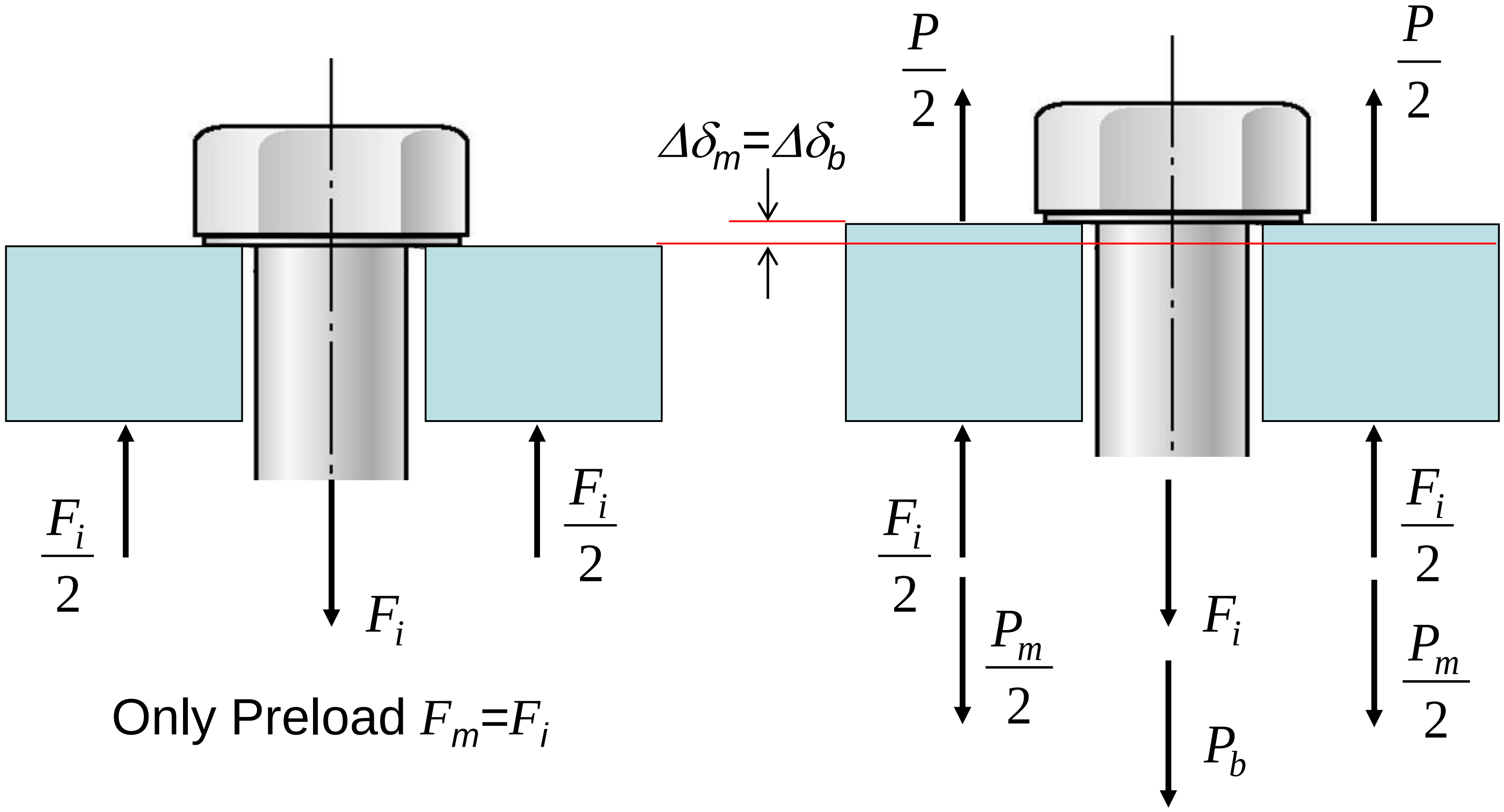
Another simpler approach is to assume that a hollow cylinder of inner diameter d and outer diamed $3d$ is compressed rather than a conical frustum.

Force Analysis of Bolted Joints in Tension

- Bolt is given a preload F_i.
- Then joint is subjected to an external force P.
- We want to know what is the force acting on the bolt and members.



FBD and deflections



Only Preload $F_m = F_i$

A load P (per bolt) is applied

Assuming members are not separated $\Delta\delta_m = \Delta\delta_b$

- P: External load on the joint (for each bolt)
- F_i:preload due to tightening which exist before P is applied
- P_b:portion of P taken by the bolt
- P_m:portion of the load taken by the members (due to release of preload on members)
- F_b:resultant bolt load
- F_m:resultant member load

$$\Delta\delta_b = \frac{P_b}{k_b}$$

$$\Delta\delta_m = \frac{P_m}{k_m}$$

If members are not separated $\Delta\delta_m = \Delta\delta_b$

$$\Delta\delta_b = \frac{P_b}{k_b} = \frac{P_m}{k_m} = \Delta\delta_m \Longrightarrow P_m = \frac{k_m P_b}{k_b}$$

From static equilibrium $P = P_b + P_m \Longrightarrow P = P_b + \frac{k_m P_b}{k_b}$

$$P_b = \frac{k_b P}{k_b + k_m}$$

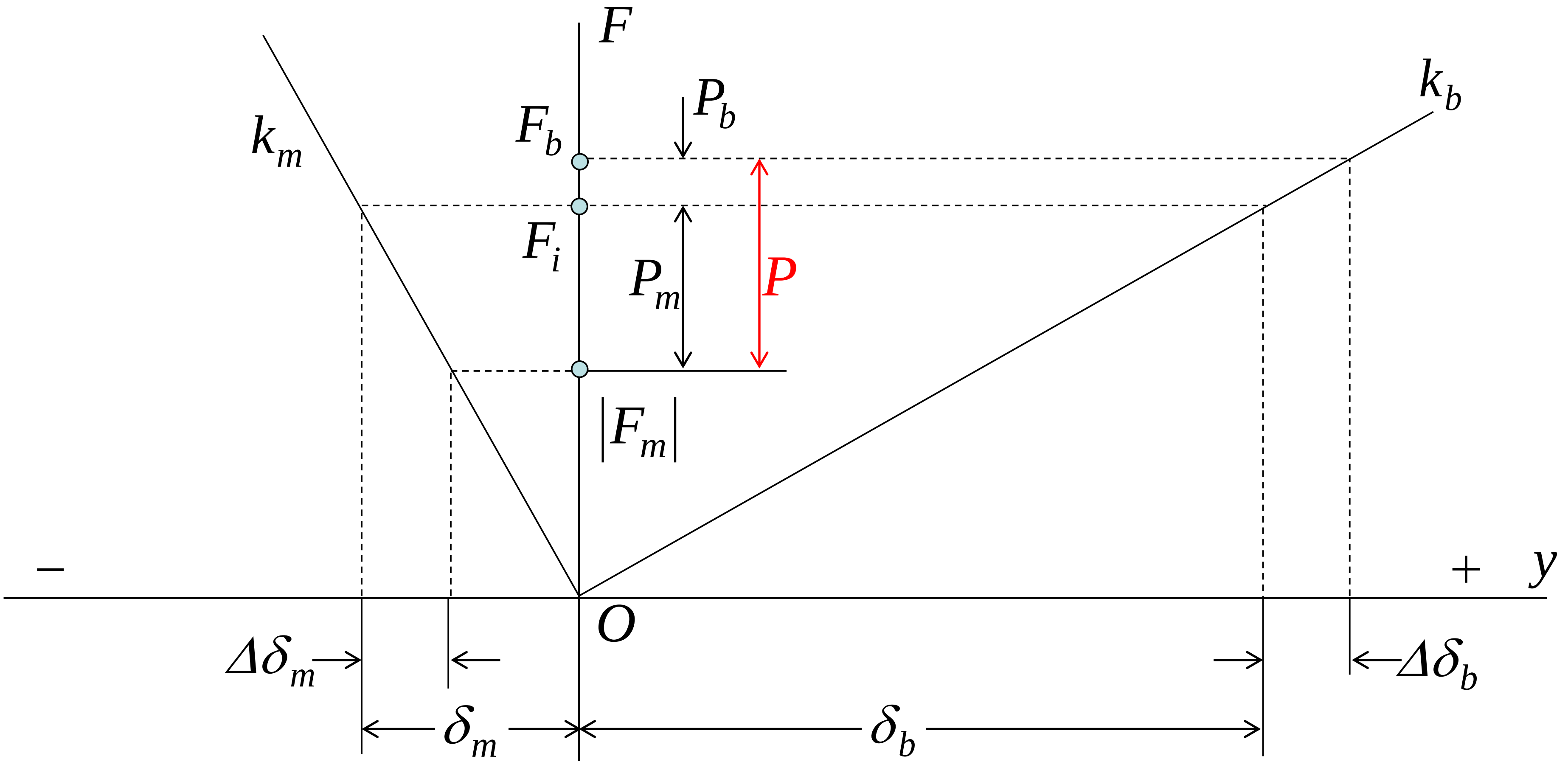
$$P_m = \frac{k_m P}{k_b + k_m}$$

Joint constant, C:

$$C = \frac{k_b}{k_b + k_m}$$

$$F_b = P_b + F_i = \frac{k_b P}{k_b + k_m} + F_i$$ as long as $F_m < 0$

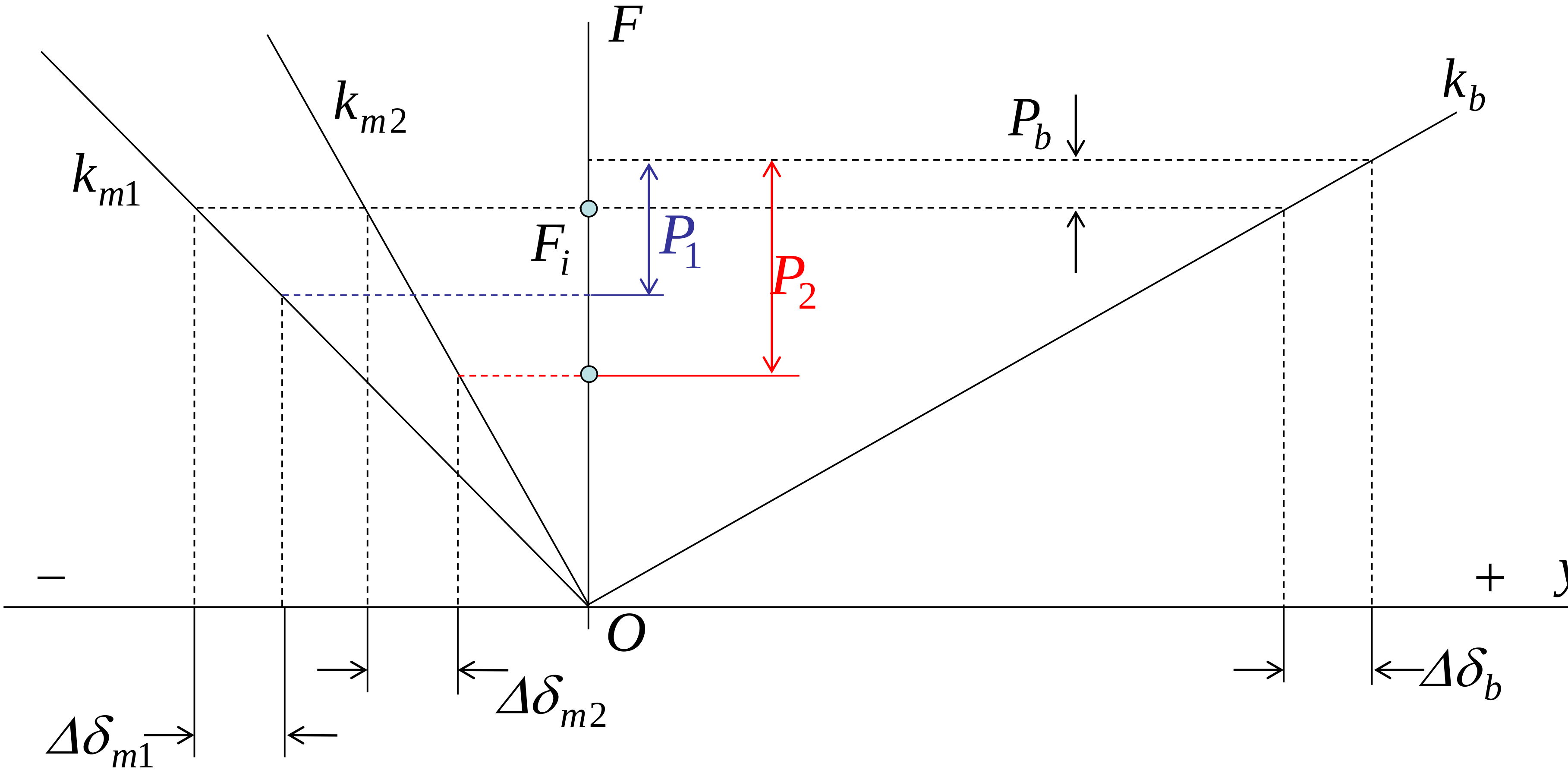
$$F_m = P_m - F_i = \frac{k_m P}{k_b + k_m} - F_i$$ as long as $F_m < 0$



$$\Delta\delta_m = \Delta\delta_b$$

As k_m/k_b decreases share of the load carried by the bolt (P_b) decreases.

In order to prevent separation F_i must be sufficiently high, otherwise F_m can become greater than or equal to zero.



$F_b = P_b + F_i = CP + F_i$ as long as $F_m < 0$

$F_m = P_m - F_i = (1-C)P - F_i$ as long as $F_m < 0$

Torque Requirements

- To develop required preload how much torque should be applied on the bolt?
- Remember for power screws:

$$T = \frac{F_i d_m}{2}\left(\frac{\ell + \pi\mu d_m \sec\alpha}{\pi d_m - \mu\ell\sec\alpha}\right) + \frac{F_i \mu_c d_c}{2} \quad \text{and} \quad \tan\lambda = \frac{\ell}{\pi d_m}$$

dividing numerator and denominator by πd_m, and using $d_c=1.25d$

$$T = \underbrace{\left[\frac{d_m}{2d}\left(\frac{\tan\lambda + \mu\sec\alpha}{1-\mu\tan\lambda\sec\alpha}\right) + 0.625\mu_c\right]}_{K} F_i d$$

K : Torque coefficient

$$T = KF_i d$$

We use $K=0.2$ when the bolt condition is not stated. If the bolt condition is one of those given in the table, we use that.

Bolt Condition	*K*
Nonplated, black finish	0.30
Zinc-plated	0.20
Lubricated	0.18
Cadmium-plated	0.16
With Bowman Anti-Seize	0.12
With Bowman-Grip nuts	0.09

This method can not produce accurate preloading. If possible we can measure bolt elongation and use

$$\delta = \frac{F_i \ell}{AE}$$

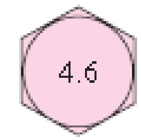
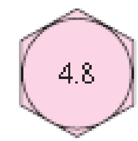
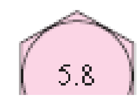
Strength Specifications

- Metric bolts, screws, nuts, studs are classified according to property class (For SAE bolts we use the term "grade")
- Proof load: Maximum force that a bolt can withstand without acquiring permanent set.
- Proof strength: Limiting value of the stress determined by using proof load and the tensile stress area.

$$S_p = \frac{F_p}{A_t}$$

Table 8–11

Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs*

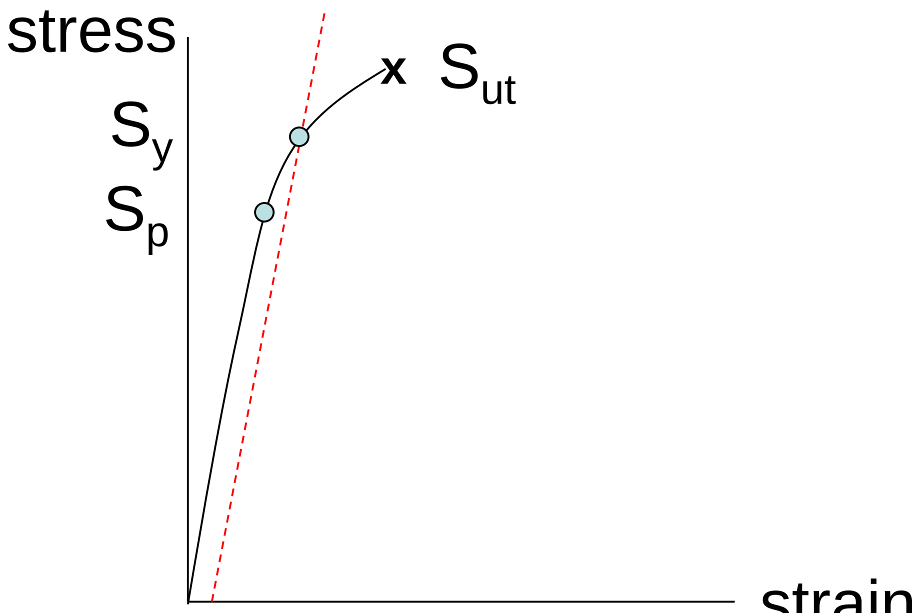
Property Class	Size Range, Inclusive	Minimum Proof Strength,† MPa	Minimum Tensile Strength,† MPa	Minimum Yield Strength,† MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	
4.8	M1.6–M16	310	420	340	Low or medium carbon	
5.8	M5–M24	380	520	420	Low or medium carbon	
		………………				
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	

An indication of min. yield strength

Gives (roughly) min. tensile strength

Unless otherwise specified we can take $S_p = 0.85 S_y$

Yield strength is based on 0.2% permanent deformation so it is usually slightly higher than proof strength.



Typical stress-strain curve for bolt materials:

- there is no clear yield point
- curve is smooth upto fracture

Bolt Preload:

Because of the preload, most of the external load is carried by the members. It is recommended that

$$F_p = A_t S_p$$

$$F_i = \begin{cases} 0.75 F_p & \text{reused connections} \\ 0.90 F_p & \text{permanent connections} \end{cases}$$

Static Loading Requirements

1. Bolt preload F_i must be high enough to prevent separation under external force P
2. $F_b = F_i + P_b$ should not exceed proof load
3. The bolt must not yield (or fracture) during tightening

<u>Requirement 1 (no separation) :</u> At separation $F_m = 0$

$$F_m = (1 - C)P - F_i$$

let P_0 be the external force required for separation. (Then $F_b = P_0$)

$$(1 - C)P_0 - F_i = 0$$

let n be the factor of safety against separation

$$n = \frac{P_0}{P} \Rightarrow P_0 = nP \Rightarrow (1 - C)nP - F_i = 0 \Rightarrow n = \frac{F_i}{P(1 - C)}$$

If $n > 1$ then joint is safe against separation.

Requirement 2 : (S_p=0 not exceeded)

$$F_b = CP + F_i$$

divide both sides by A_t

$$\frac{F_b}{A_t} = \frac{CP}{A_t} + \frac{F_i}{A_t} \qquad \sigma_b = \frac{F_b}{A_t}$$

F_i has already been fixed. To take into account uncertainties in loading introduce load factor n and use nP rather than P.

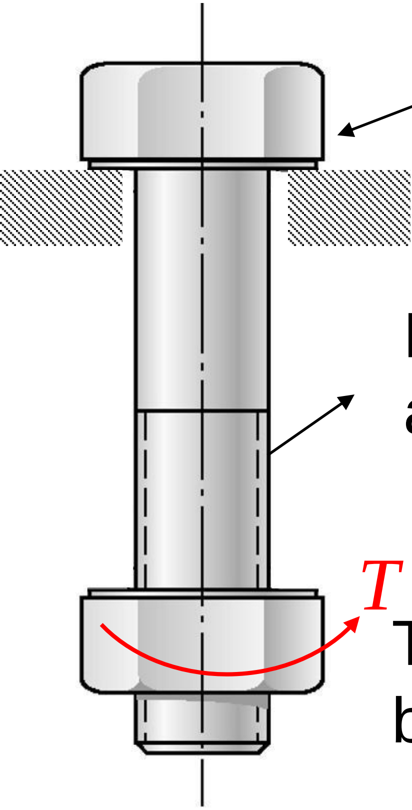
$$\sigma_b = S_p \quad \Rightarrow \quad S_p = \frac{C(nP)}{A_t} + \frac{F_i}{A_t} \quad \Rightarrow \quad n = \frac{S_p A_t - F_i}{CP}$$

If $n>1$ proof strength of the bolt has not been exceeded.

n is like a factor of safety but we applied it to load P only since F_i can be accurately determined.

Requirement 3 : Safety during tightening

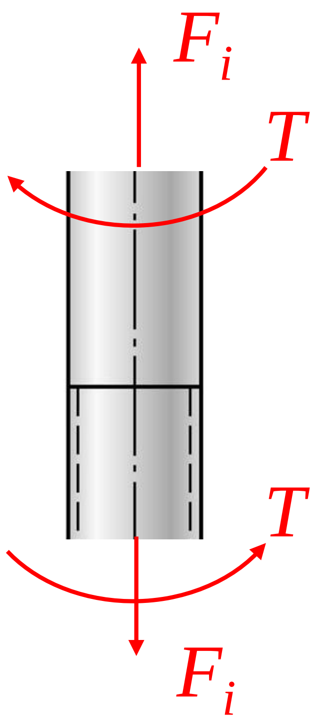
Note that the bolt is in a state of combined axial loading and torsion at the final stage of tightening.



Cap is fixed by the friction forces acting on its bottom surface.

Body is stretched upto an axial load of F_i.

Tigthening torque T is being applied on the nut.



$$\sigma = \frac{F_i}{A_t}$$

$$\tau = \frac{Tr}{J}$$

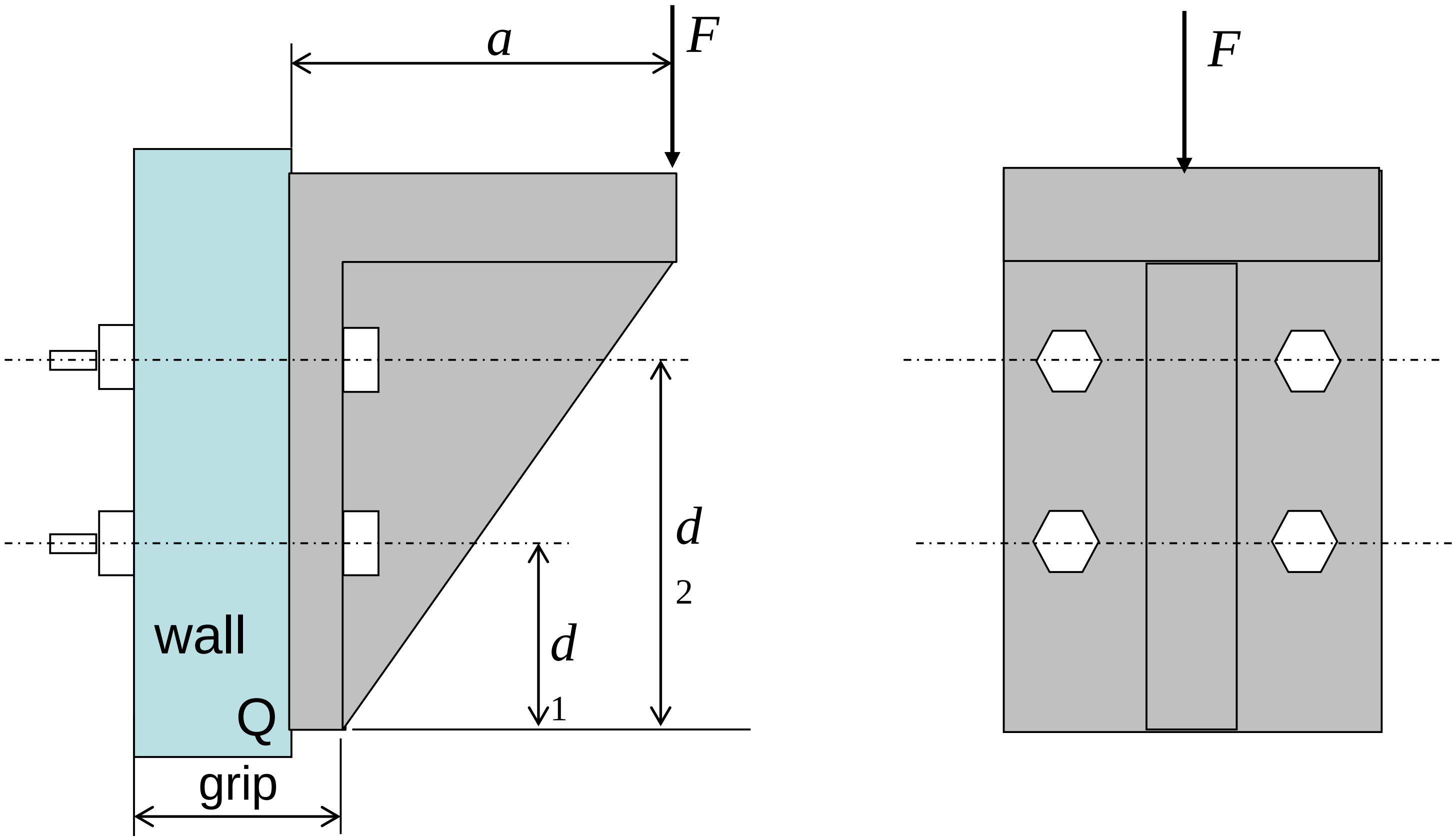
$$r = \sqrt{\frac{A_t}{\pi}}$$

$$J = \frac{\pi r^4}{2}$$

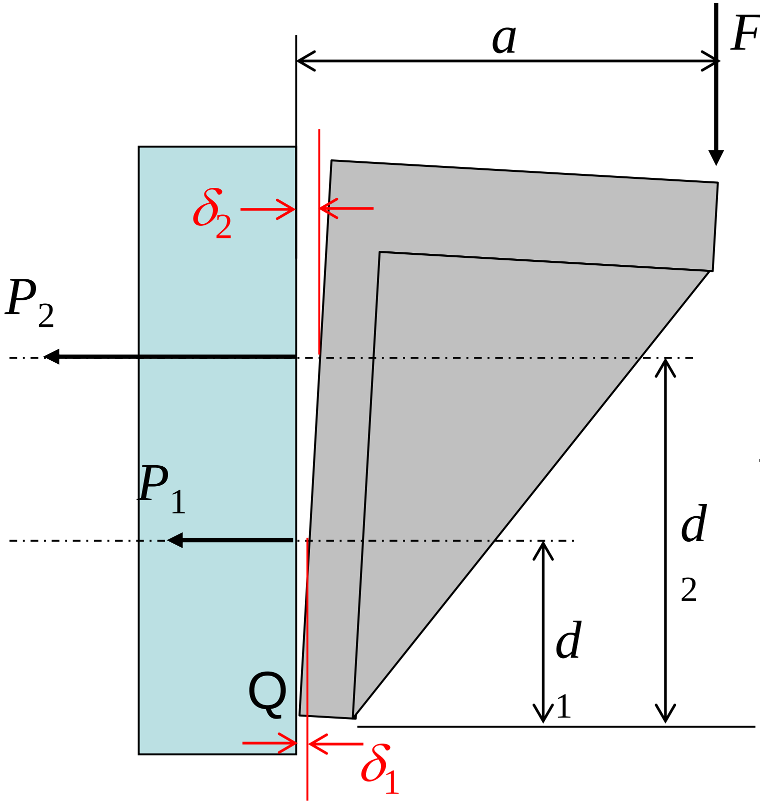
MSST or DET can be used to determine factor of safety.

Analysis of a Bracket

- Sometimes, the total load on a joint is not distributed equally on the bolts.
- For example consider the bracket below:



- It is assumed that the bracket is rigid and tends to make a tipping motion about point Q.
- The external loads applied on the bracket at the locations of bolts are P_1 and P_2.



$$P_1 = k_1\delta_1 \quad P_2 = k_2\delta_2$$

$$\frac{P_1}{P_2} = \frac{k_1\delta_1}{k_2\delta_2} \quad \text{but} \quad \frac{d_1}{d_2} = \frac{\delta_1}{\delta_2}$$

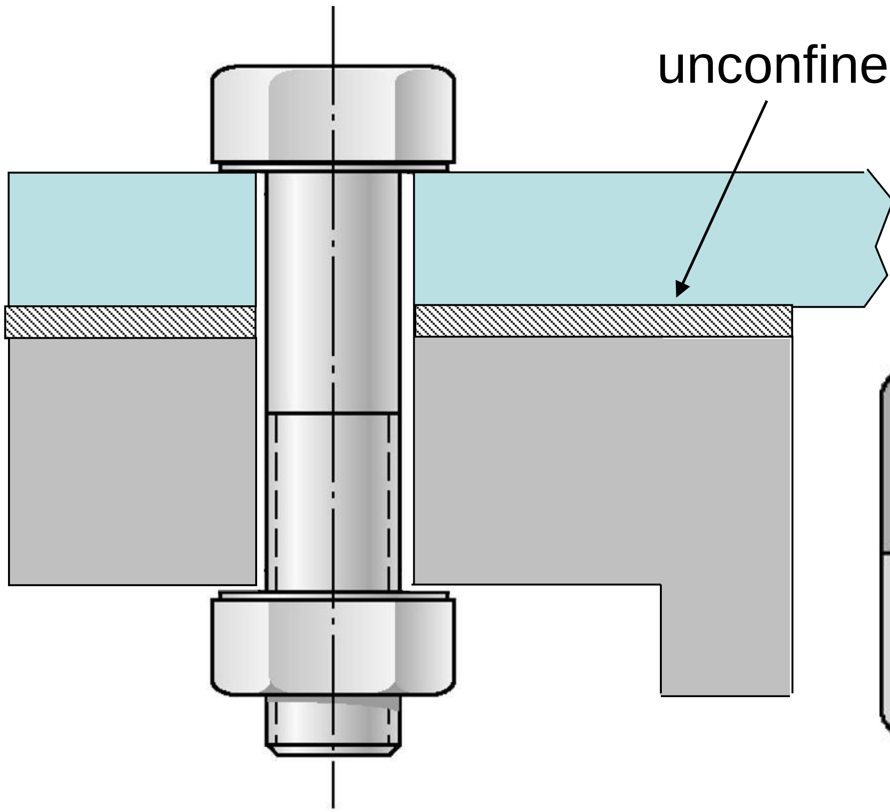
$$\frac{P_1}{P_2} = \frac{k_1 d_1}{k_2 d_2}$$ if the bolts are identical $k_1 = k_2$

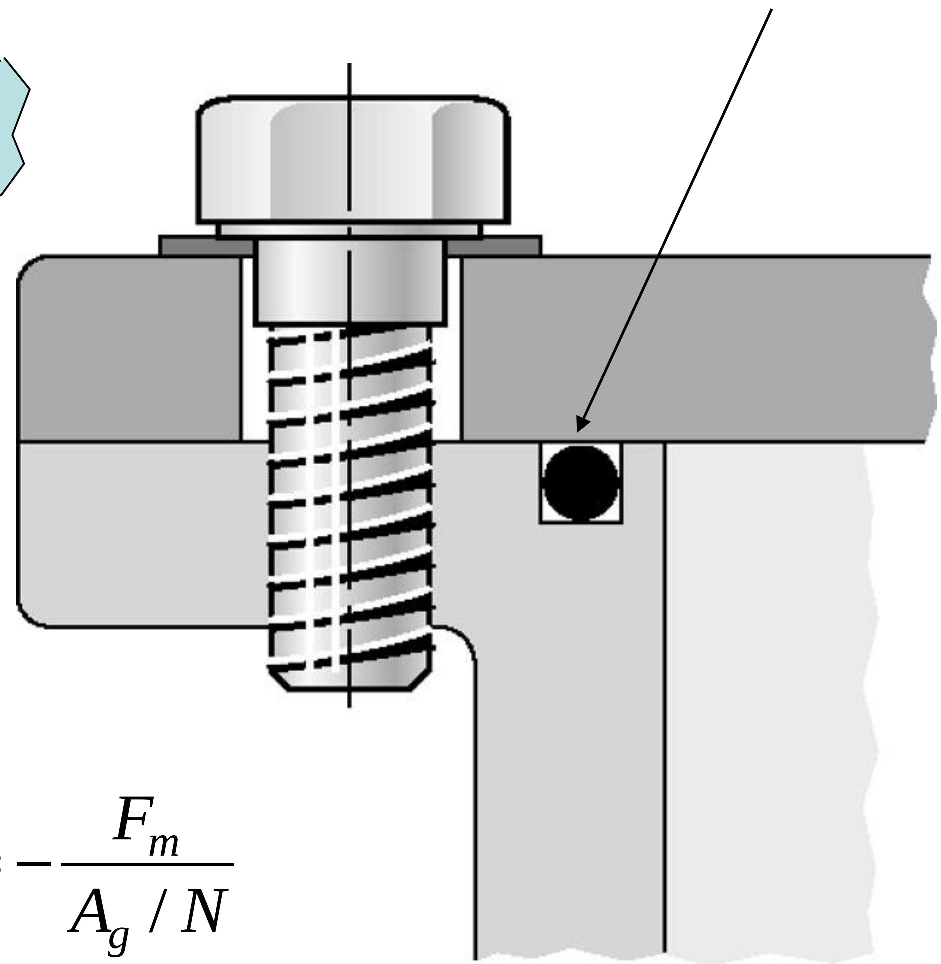
$$\frac{P_1}{P_2} = \frac{d_1}{d_2} \quad (1)$$

From equilibrium $d_1P_1 + d_2P_2 = aF$ (2)

From (1) and (2), P_1 and P_2 may be obtained. External load per bolt is obtained by dividing them by two.

Gasketed Joints





unconfined (full) gaskets require a gasket pressure p to be maintained on them, to prevent leakage.

$$p = -\frac{F_m}{A_g / N}$$

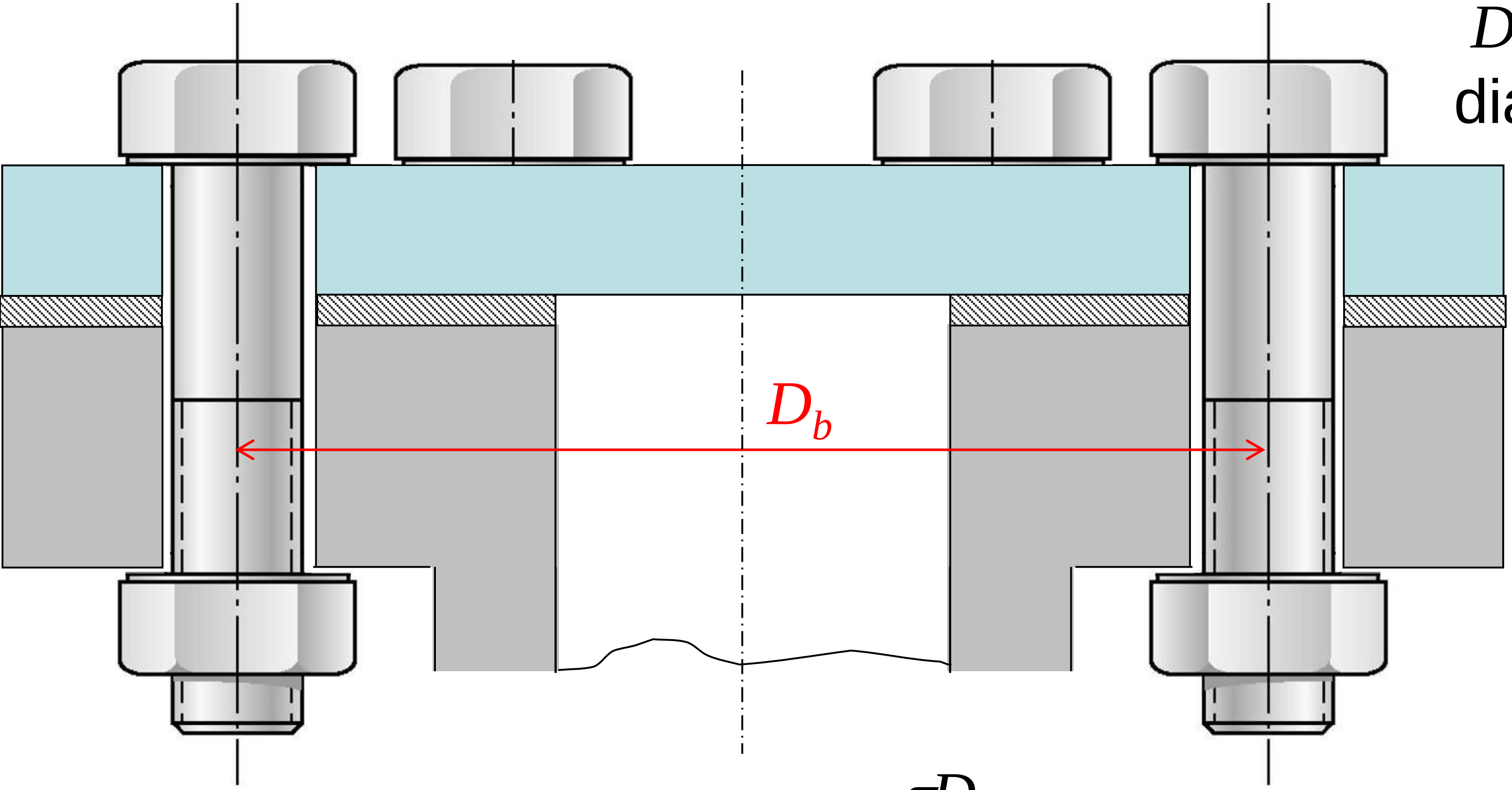
A_g Gasket Area, N Number of bolts

Introduce load factor n into member force expression

$F_m = (1-C)nP - F_i$ substituting this into above expression,

$$p = \frac{\left(F_i - (1-C)nP\right)N}{A_g}$$

D_b Bolt circle diameter



As a rule of thumb $3 \le \frac{\pi D_b}{Nd} \le 6$

Fatigue Loading

- The previously covered methods (Goodman, Soderberg) are applicable to design of bolts subjected to tensile loading.
- Fatigue strength reduction factors which are also corrected for notch sensitivity and surface finish are given in the text book.
- Fully corrected endurance limits for rolled threads are also given.

Table 8–16

Fatigue Stress-Concentration Factors K_f for Threaded Elements

SAE Grade	Metric Grade	Rolled Threads	Cut Threads	Fillet
0 to 2	3.6 to 5.8	2.2	2.8	2.1
4 to 8	6.6 to 10.9	3.0	3.8	2.3

Table 8–17

Fully Corrected Endurance Strengths for Bolts and Screws with Rolled Threads*

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi
SAE 8	$\frac{1}{4}$–$1\frac{1}{2}$ in	23.2 kpsi
ISO 8.8	M16–M36	129 MPa
ISO 9.8	M1.6–M16	140 MPa
ISO 10.9	M5–M36	162 MPa
ISO 12.9	M1.6–M36	190 MPa

*Repeatedly-applied, axial loading, fully corrected.

Let the external load per bolt on the joint vary between P_{min} and P_{max}.

$$(F_b)_{\max} = CP_{\max} + F_i$$

$$(F_b)_{\min} = CP_{\min} + F_i$$

$$P_a = \frac{P_{\max} - P_{\min}}{2} \qquad P_m = \frac{P_{\max} + P_{\min}}{2}$$

Alternating and mean stresses on the bolt:

$$\sigma_a = \frac{(F_b)_{\max} - (F_b)_{\min}}{2A_t} = \frac{C(P_{\max} - P_{\min})}{2A_t} = \frac{CP_a}{A_t}$$

$$\sigma_m = \frac{(F_b)_{\max} + (F_b)_{\min}}{2A_t} = \frac{C(P_{\max} + P_{\min})}{2A_t} + \frac{F_i}{A_t} = \frac{CP_m}{A_t} + \frac{F_i}{A_t}$$

To write the equation of load line, one needs to make an assumption. Here assume that the ratio P_m/P_a be a constant. In other words P_{max} and P_{min} may increase or decrease by the same factor.

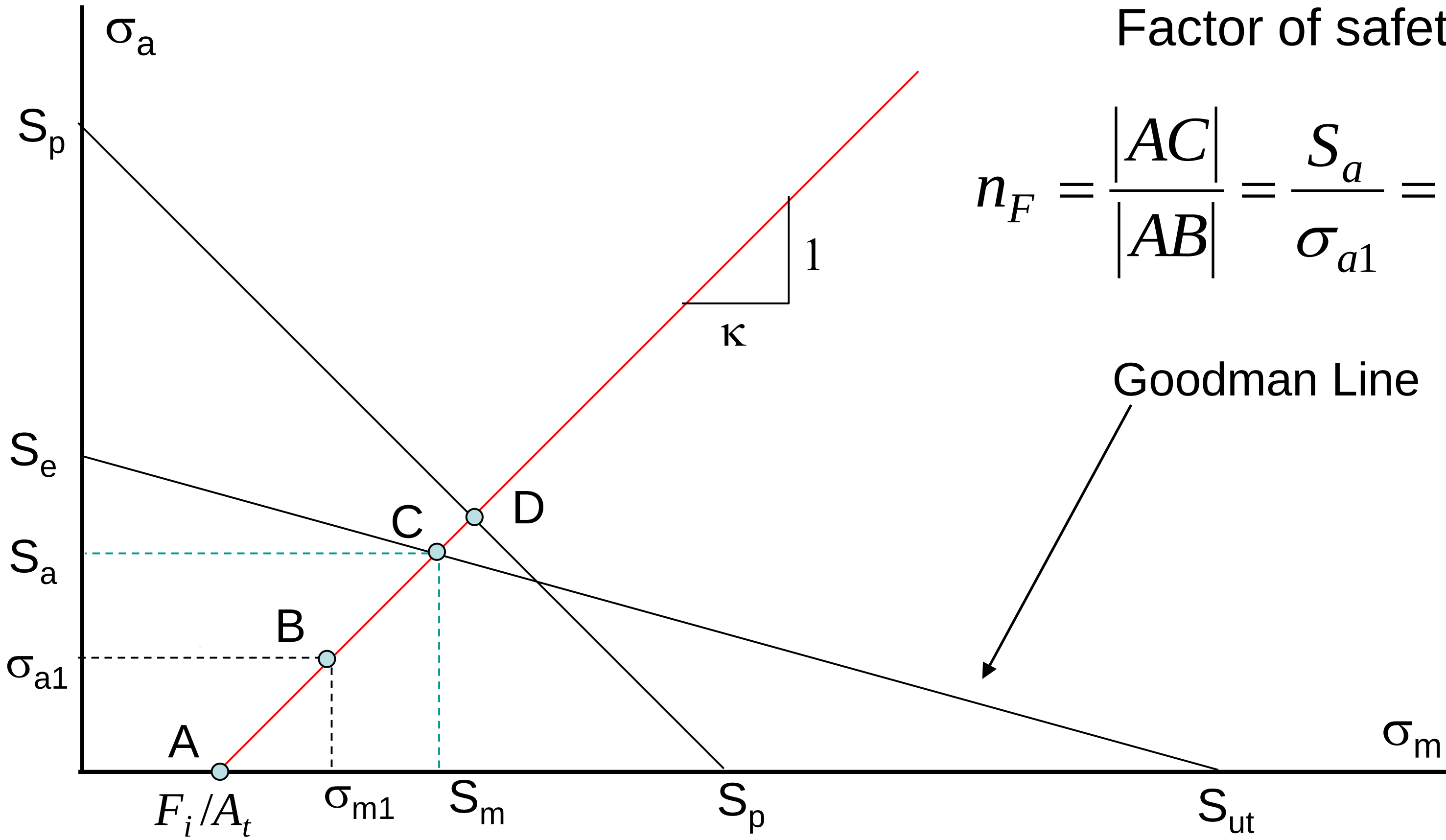
$$\frac{P_m}{P_a} = \kappa \quad \Longrightarrow \quad P_m = \kappa P_a$$

$$\sigma_m = \frac{CP_m}{A_t} + \frac{F_i}{A_t} = \frac{C\kappa P_a}{A_t} + \frac{F_i}{A_t} = \kappa\sigma_a + \frac{F_i}{A_t}$$ Equation of load line

Note that load line has a slope of $1/\kappa$ and it does not pass through the origin

Note that if P_{min}=0 then κ=1 and load line has a 45 deg. slope.

$$\kappa = \frac{P_m}{P_a} = \frac{(P_{\max} + P_{\min})/2}{(P_{\max} - P_{\min})/2} = \frac{P_{\max} + 0}{P_{\max} + 0} = 1$$



Factor of safety for point B:

$$n_F = \frac{|AC|}{|AB|} = \frac{S_a}{\sigma_{a1}} = \frac{S_m - F_i / A_t}{\sigma_{m1} - F_i / A_t}$$

We need to find coordinates of point C(S_m,S_a) which is on both Goodman and load lines

$$S_m = \kappa S_a + \frac{F_i}{A_t} \quad \textbf{(1)} \qquad \frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad \textbf{(2)}$$

Eliminating S_m one can determine S_a as $S_a = \dfrac{S_{ut} - F_i / A_t}{\kappa + S_{ut} / S_e}$

One also needs to check proof strength either by using, $n_{P1} = \dfrac{|AD|}{|AB|}$

or $n_{P2} = \dfrac{S_p}{\sigma_{\max}} = \dfrac{S_p}{\sigma_{m1} + \sigma_{a1}}$