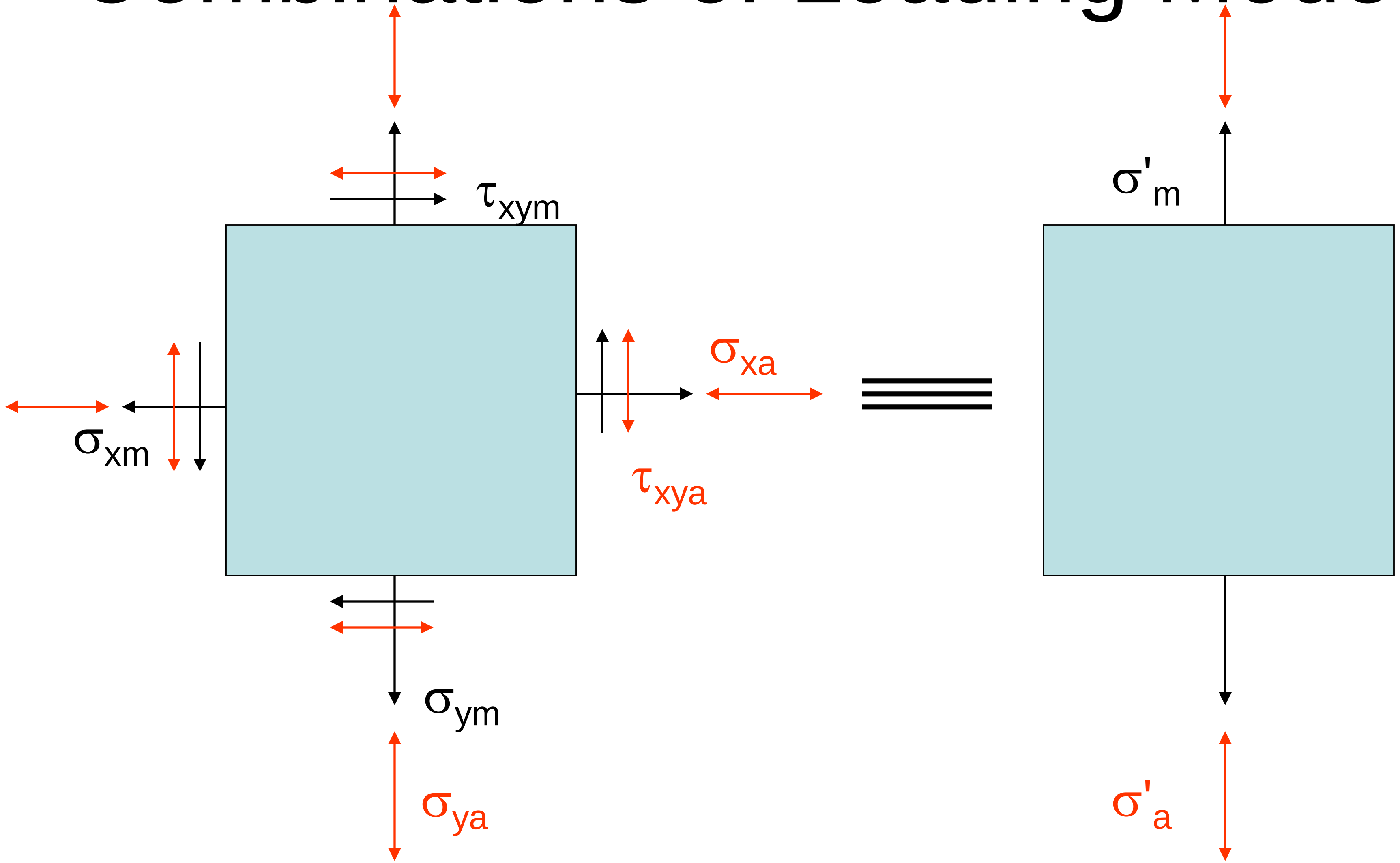
Fatigue Failure II

Combinations of Loading Modes

- In many cases stresses acting on machine elements are multiaxial stresses.
- Since most fatigue failures starts on the surface we consider biaxial case rather than full 3D.
- There are various approaches to deal with multiaxial fatigue. In this course, we adopt the simple approach of reducing the biaxial case to uniaxial case.

Combinations of Loading Modes



Combinations of Loading Modes

- σ'_m and σ'_a are equivalent stresses. We use Von-Mises stress as the equivalent stress.
- $\sigma'_m = (\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2 + 3\tau_{xym}^2)^{1/2}$ (1)
- $\sigma'_a = (\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2 + 3\tau_{xya}^2)^{1/2}$ (2)
- If principal stresses are given, shear stress is taken as zeroin the equations above.

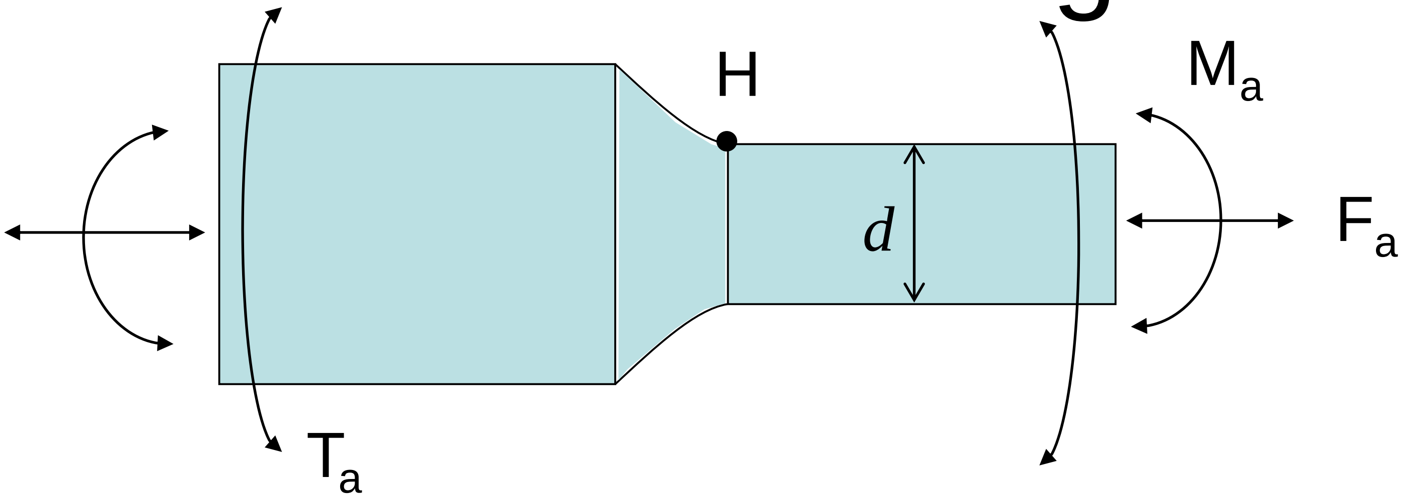
Combinations of Loading Modes

- Once the equivalent stresses are obtained we may use Goodman or Soderberg approach as if we had uniaxial case.
- Note that in these equations, it is assumed that stresses are in phase,i.e. they reach their maximum and minimum values at the same time.

Combinations of Loading Modes

- In biaxial case, we apply fatigue stress concentration factors K_f to alternating components of normal and/or shear stresses and then, take endurance limit modifying factor k_e=1.
- Similarly, we can use $1/k_b$ to amplify stresses, rather than using k_b to reduce endurance limit.

Combinations of Loading Modes



At point H which is the critical point, we have

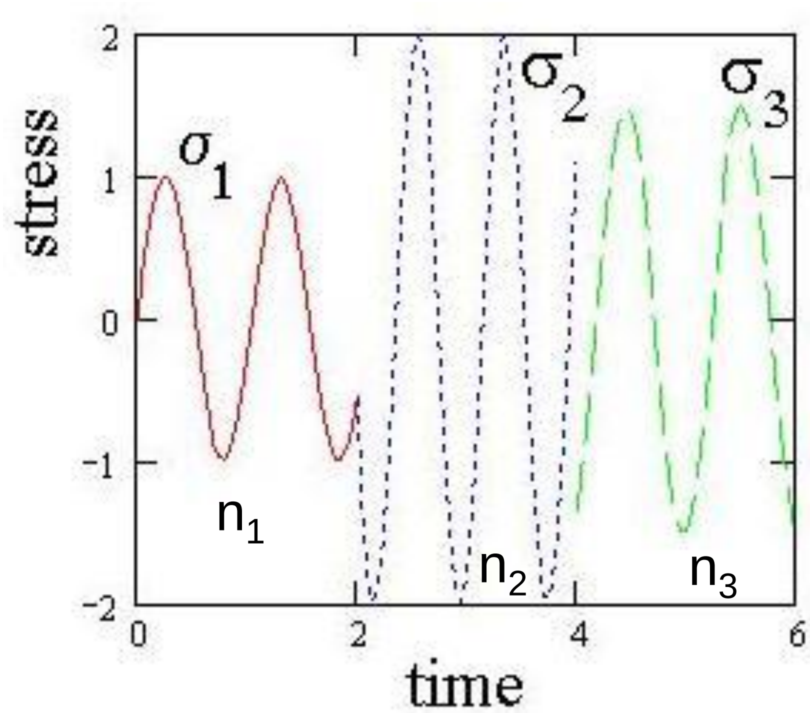
$$\sigma_{a,\text{axial}} = K_{f,\text{axial}} \frac{4F_a}{\pi d^2} \quad \sigma_{a,\text{bending}} = \frac{1}{k_b} K_{f,\text{bending}} \frac{32M_a}{\pi d^3}$$

$$\sigma_{xa} = \sigma_{a,\text{axial}} + \sigma_{a,\text{bending}} \quad \tau_{a,\text{torsion}} = \frac{1}{k_b} K_{f,\text{torsion}} \frac{16T_a}{\pi d^3}$$

Equations above should be used in eq.(2) to find equivalent alternating stress.

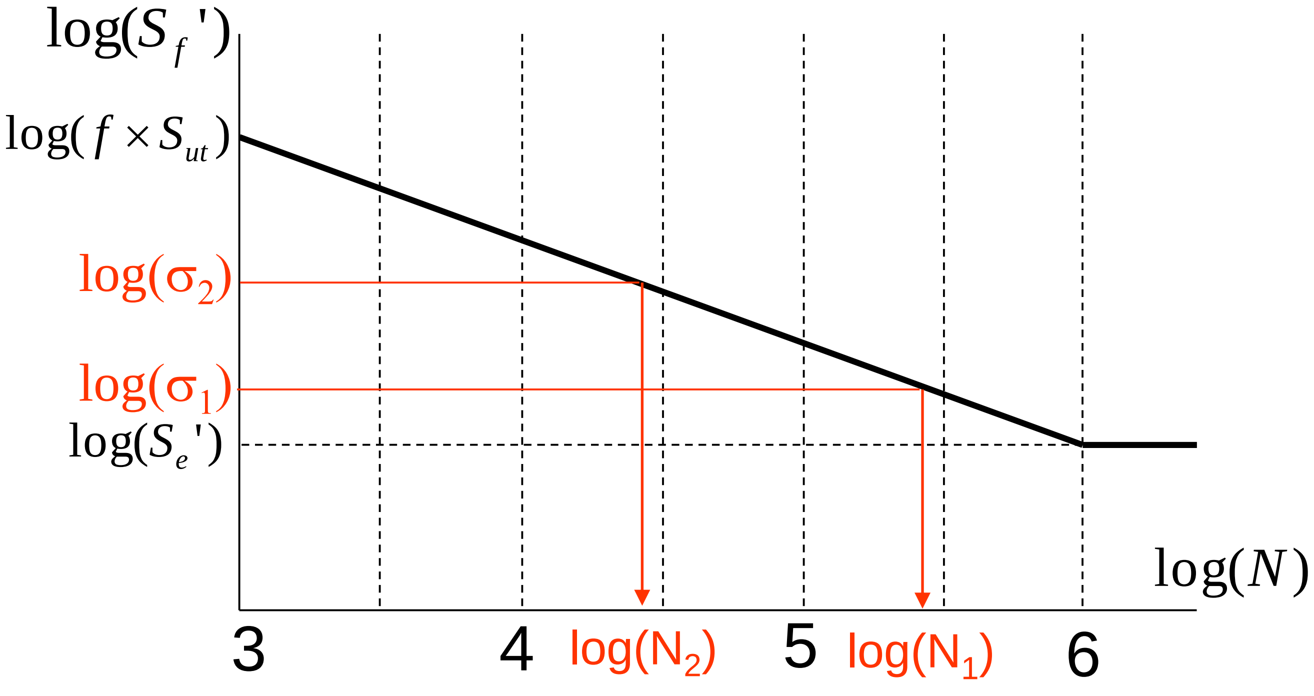
Cumulative Fatigue Damage

- If stress cycles with σ_{max}>S_e are applied, fatigue life (N) for another stress level and endurance limit decreases. To predict this reduction in life there are two approaches.
  - Miner's Rule
  - Manson's Rule



Cumulative Fatigue Damage

- stress σ_i is applied n_i cycles to the specimen, N_i is the life corresponding to stress σ_i.



Miner's Rule

- According to Miner's Rule;

$$\sum \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots\cdots + \frac{n_m}{N_m} = C \qquad 0.7 \leq C \leq 2.2$$

- C is an experimentally determined constant.
- Unless otherwise is specified we take C=1

Miner's Rule

- Miner's Rule have deficiencies
  - It does not account for the order in which the stresses are applied, hence ignores any stress less than the initial endurance limit.
  - If an attempt is made to correct endurance limit by using Miner's rule, than Sut will also become lowered, but this is not verified by the experiments.

Manson's Rule

- Manson's rule does not have the deficiencies of Miner's Rule
- In Manson's rule, S-N diagram is updated in the same historical order with the applied stresses.
- S_{ut} is kept constant and the new S-N line is drawn to pass through the point given by applied stress and remaining life.

Manson's Rule