CHAPTER IV
Fatigue Failure

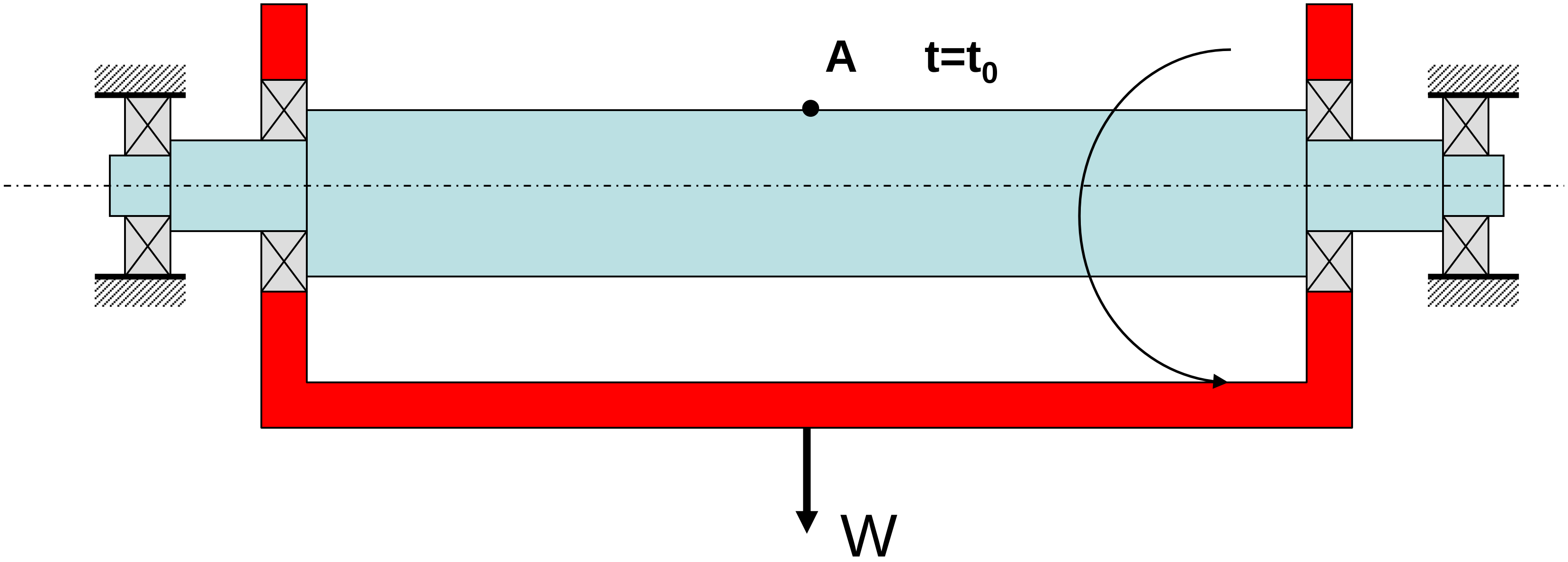
Introduction

- In fatigue loading, stresses at a given point in the material change as a function of time in a cyclic manner.
- Whereas in static loading, loads are applied gradually giving sufficient time for strain to develop. Once loading is finished, magnitude, direction, point of application of loads do not change. Hence, stresses at a given point in the material do not change.

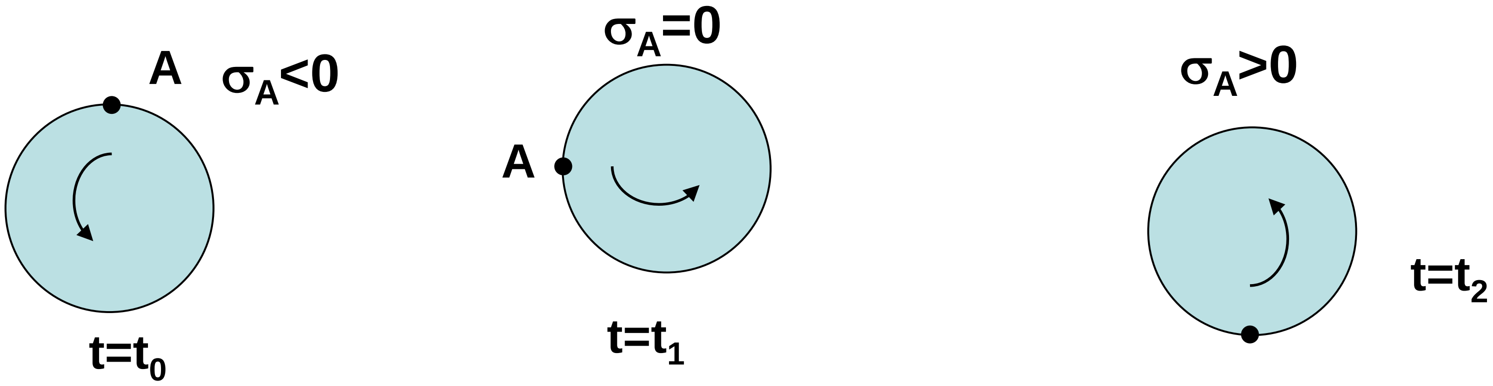
Introduction

- A typical case in fatigue loading is that of a rotating shaft subjected to transverse loads with constant magnitude and fixed direction.
- Fatigue loading arises due to rotation of the shaft.

Introduction



As the shaft rotates, stresses at point A are reversed. (compression-tension)



Typical Load Variations

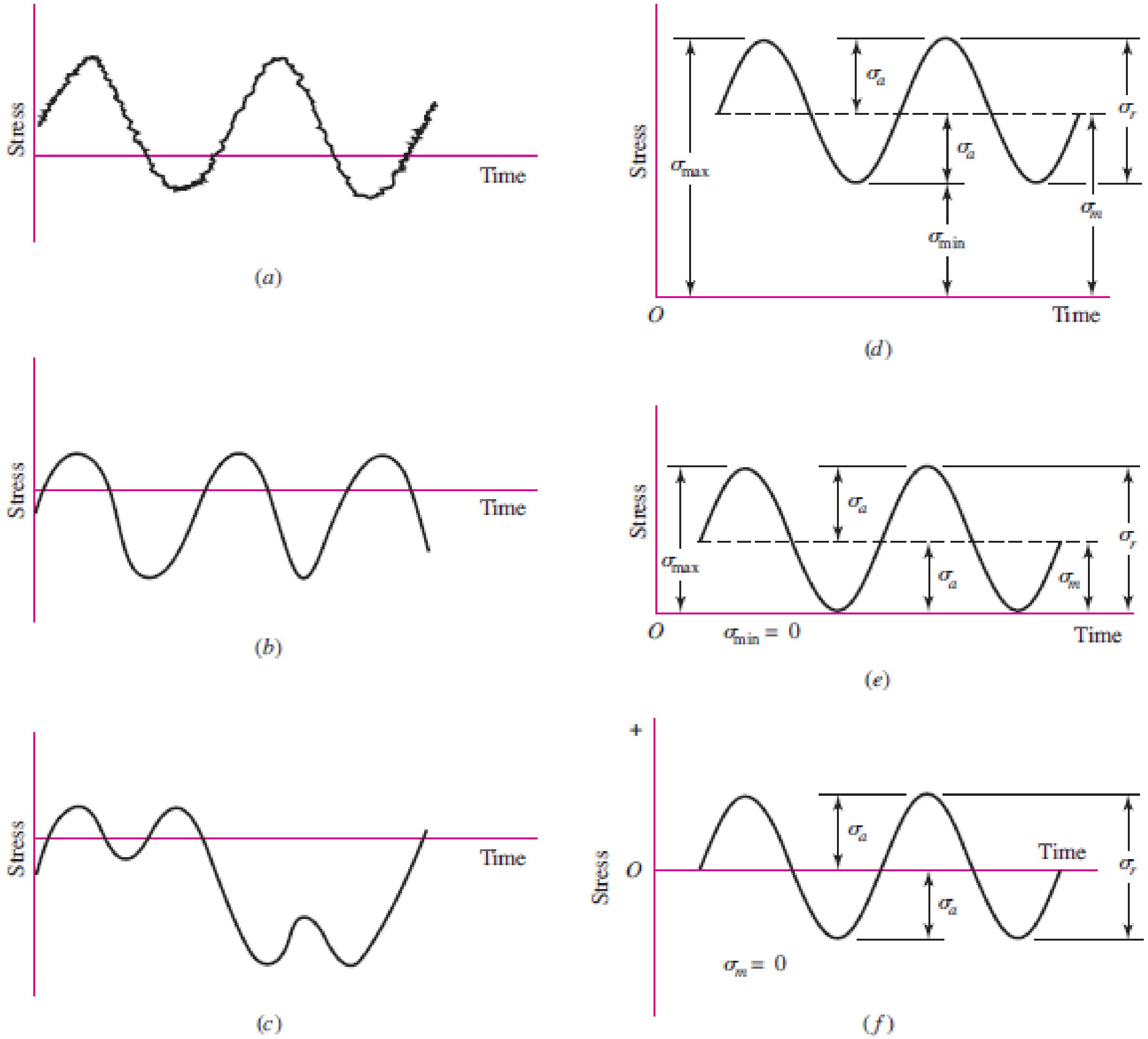
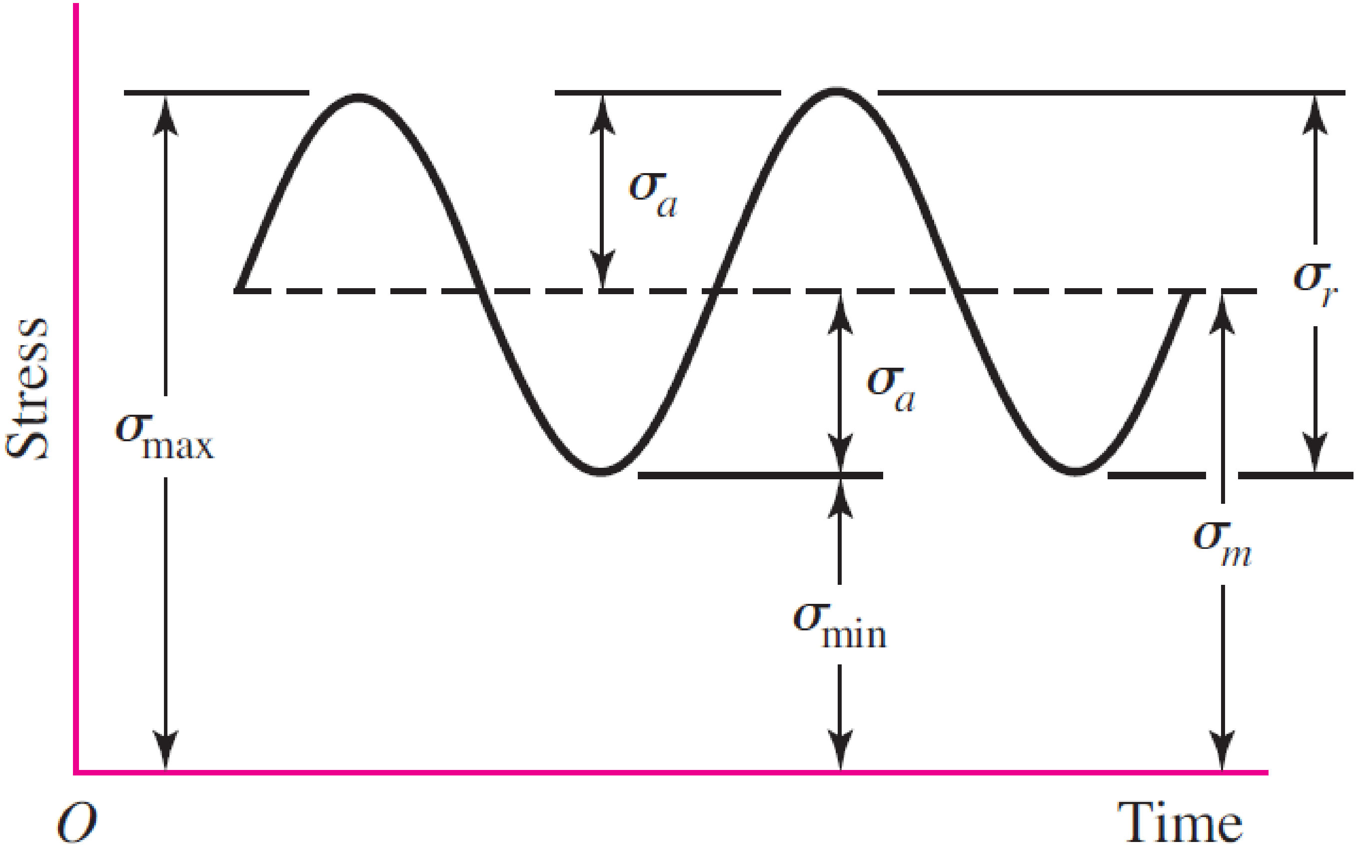


Figure 6–23

Some stress-time relations: (*a*) fluctuating stress with high-frequency ripple; (*b* and *c*) nonsinusoidal fluctuating stress; (*d*) sinusoidal fluctuating stress; (*e*) repeated stress; (*f*) completely reversed sinusoidal stress.

Fluctuating Stress

Mean Stress and alternating stress (stress amplitude)



(*d*)

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Onset of Failure

- Under static uniaxial loading, Ultimate Tensile Strength must be reached to break a member. (fracture)
- Under fatigue loading, fracture occurs although stress σ_{max} is below UTS, (frequently even below yield strength) after this stress has been applied many times.
- This kind of failure is called fatigue failure.

Fatigue Failure

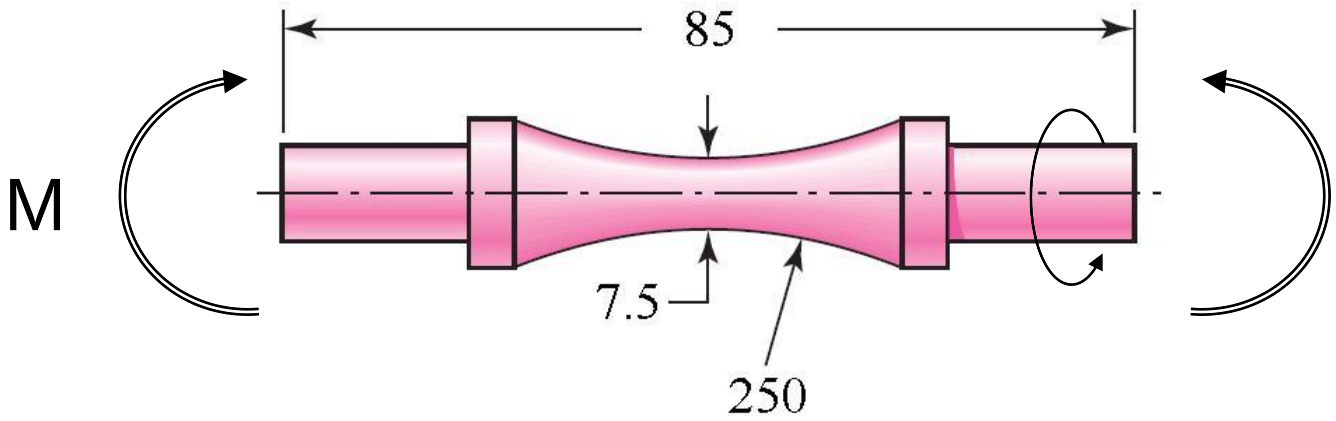
- Fatigue failures at relatively low stresses but high number of loading cycles, occur due to cracks.
- Cracks initiate where stresses are relatively high at a section (or there may be crack like defects in the material), they propagate slowly for many cycles of loading, and at some point when the crack reaches a critical size sudden fracture occurs.

Fatigue Failure

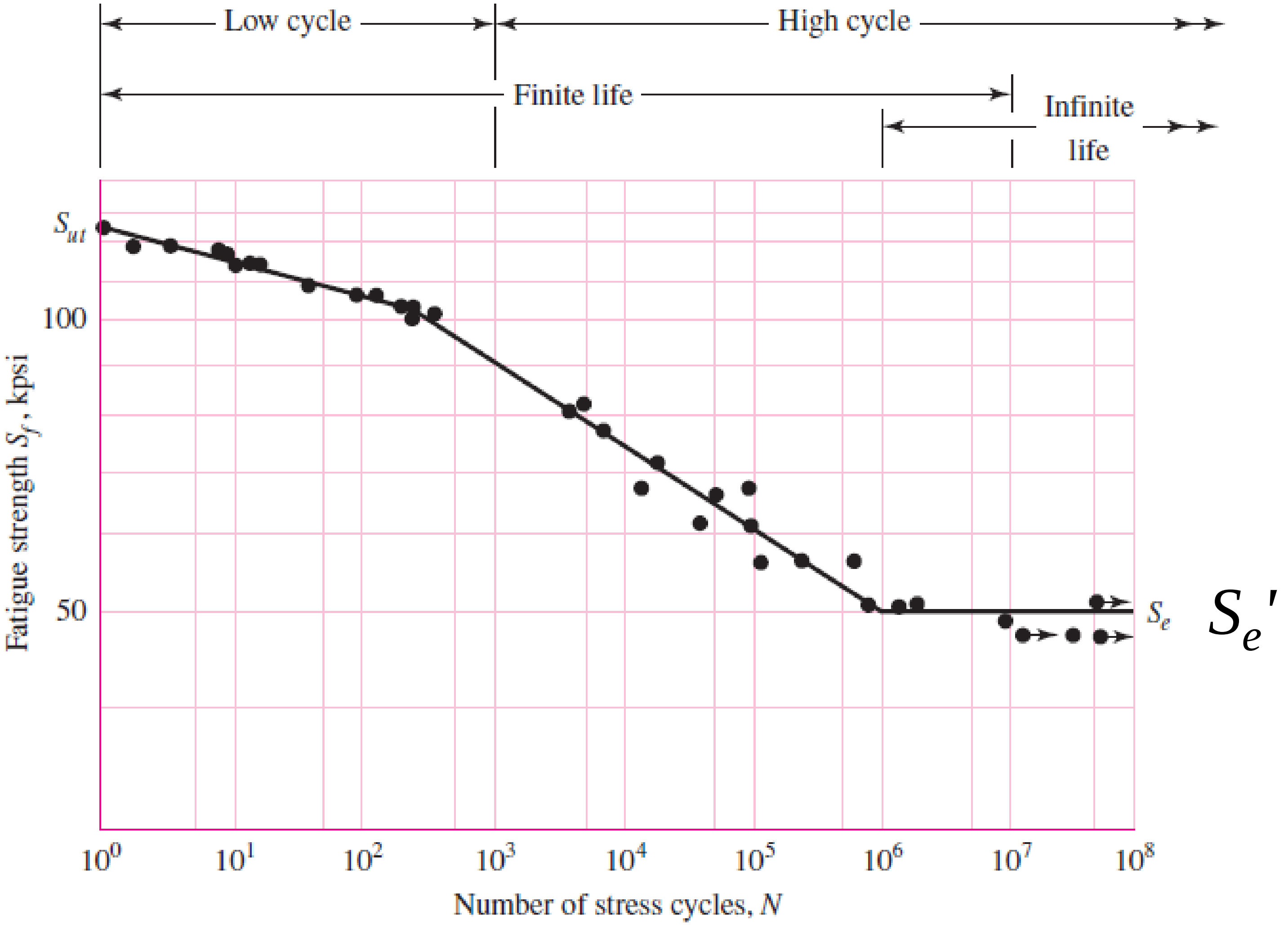
- Fatigue fracture surfaces have typical appearances.
- Slow crack propagation region (smooth) and sudden fracture region (rough) on the fracture surface can be distinguished since they have different textures.
- See figures 6.1-6.8 in your text book.
- Since fatigue failure may not give any visible warning such as excessive deflection, it can be more dangerous than static failure due to overload.

S-N Diagrams

- S-N diagrams show stress vs number of cycles until fracture.
- A common test specimen is the "rotating beam" test specimen.
- Specimens are subjected to fatigue loading and cycles are counted to destruction.



S-N Diagram



S-N Diagram for a ferrous material

S-N Diagram

- For failure, if N < 1000 we have "low cycle fatigue".This type of fatigue is associated with plastic straining rather than crack growth.
- IF N>1000 we have "high cycle fatigue".In this course high cycle fatigue will be covered.
- When the life is between 1 cycle and roughly 10 million cycles we have finite life.
- For finite life, in the range $10^3<N<10^6$, if fatigue strength S_f is referred to, corresponding number of cycles, N, must also be given.

S-N Diagram

- Beyond (roughly) 1 million cycles, S-N diagram becomes horizontal. We have infinite life. Corresponding strength is called endurance limit.
- In other words, endurance limit (fatigue limit) is the stress level, under which no failure will occur no matter how great the number of stress cycles may be.

S-N Diagram

- Note that the endurance limit of an actual member made of the same material may be different from that of the specimen for various reasons.
- For non-ferrous metals and alloys there is no endurance limit.
- For such metals, no matter how small is the applied stress, fatigue failure occurs at some large number of cycles.

S-N Diagram

- Fatigue testing is lengthy and expensive. For preliminary design and failure analysis a quick estimate of endurance limit is needed.
- In this course, based on published data we can use the following simple rules unless otherwise is stated:

$$\left.\begin{aligned} S_e' &= 0.5\,S_{ut}\,, & S_{ut} &\le 1400\,\mathrm{MPa} \\ S_e' &= 700\,\mathrm{MPa}, & S_{ut} &> 1400\,\mathrm{MPa} \end{aligned}\right\}\ \text{For steels}$$

S-N Diagram

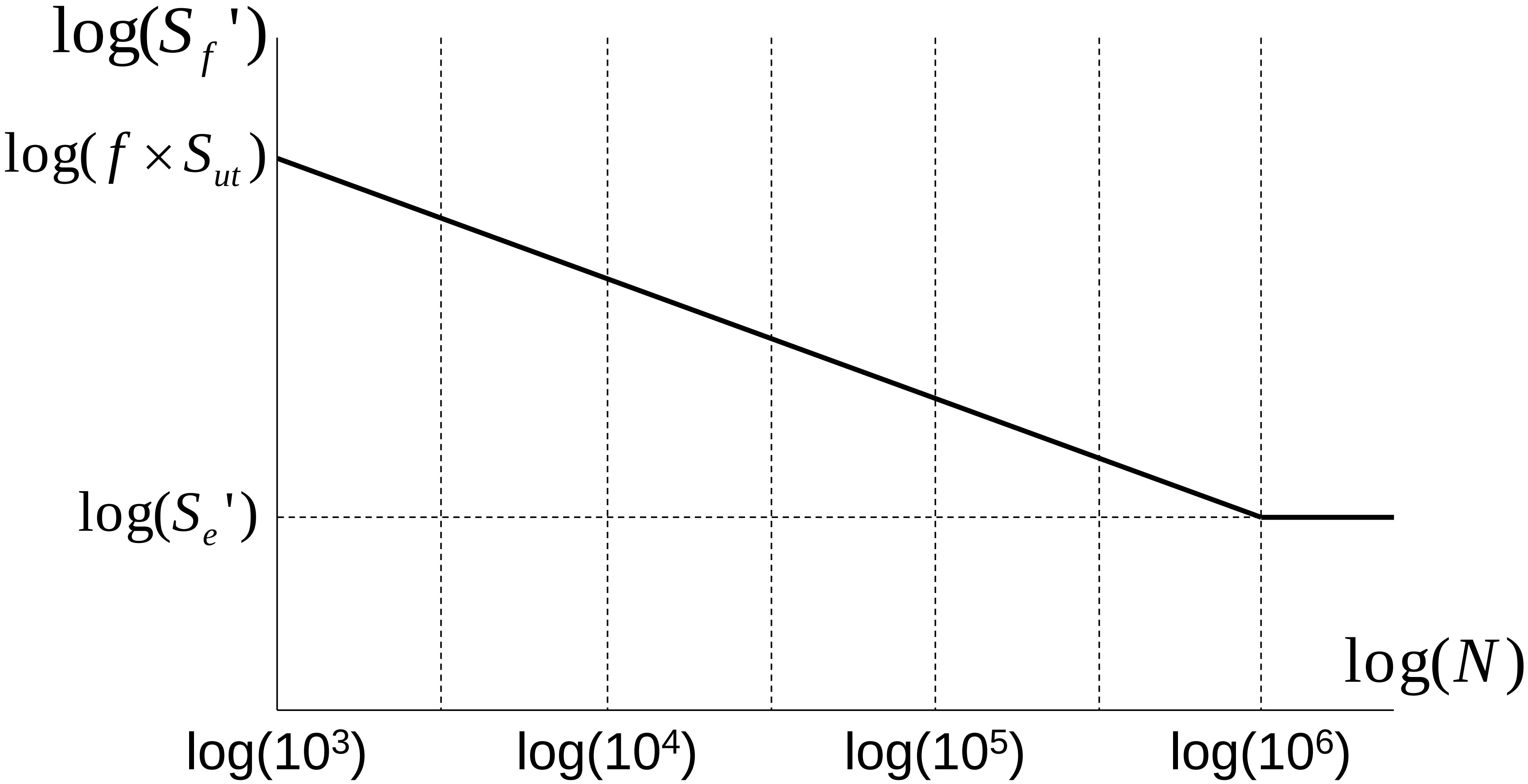
$$\left.\begin{array}{ll} S_e' = 0.45\,S_{ut}\,, & S_{ut} \le 600\,\mathrm{MPa} \\ \\ S_e' = 275\,\mathrm{MPa}, & S_{ut} > 600\,\mathrm{MPa} \end{array}\right\}\text{Cast iron and cast steel}$$

- For non-ferrous metals and alloys,fatigue strength S_f is usually specified for $N=10^8$ or $5x10^8$ cycles. For example, in case of Al and Mg alloys, S_f=30%-40% of S_{ut} for $N=10^8$ cycles.

S-N Diagram

- Note that, for steels, S_e'/S_{ut} actually depends very much on microstructure. If available, one should use actual test data. As ductility increases so does S_e'/S_{ut}.
- In high cycle fatigue, for steels, S-N diagram is approximated by using a straight line as shown below.

Approximate S-N Diagram



We can take *f* as 0.8~0.9

Approximate S-N Diagram

- Equation of the straight line for finite life region is $\log(S_f') = b\log(N) + C$

- To find parameters b and C we can write

$$\log(f \times S_{ut}) = b\log(10^3) + C$$

$$\log(S_e') = b\log(10^6) + C$$

- Solving for b and C we get

$$b = -\frac{1}{3}\log\left(\frac{\left(f \times S_{ut}\right)}{S_e'}\right) \qquad C = \log\left(\frac{\left(f \times S_{ut}\right)^2}{S_e'}\right)$$

Approximate S-N Diagram

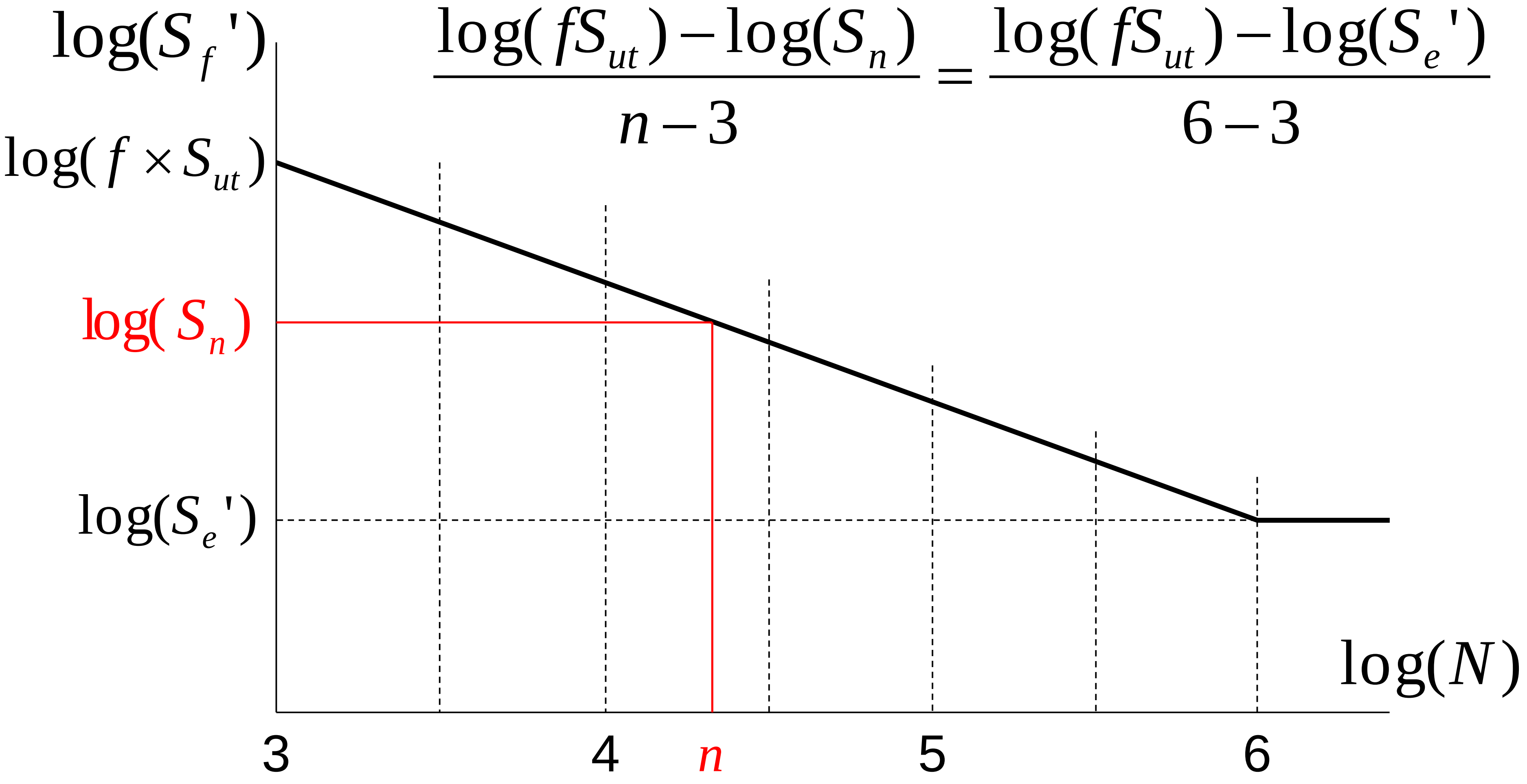
- Then we can write

$$S_f' = 10^C N^b \;,\; 10^3 \le N \le 10^6$$

$$N = 10^{-C/b} \left(S_f'\right)^{1/b} \;,\quad 10^3 \le N \le 10^6$$

- Note that instead of using the formulae above, we can use similar triangles to find S_f corresponding to an N, and vice versa.

Approximate S-N Diagram



$$\frac{\log(fS_{ut}) - \log(S_n)}{n-3} = \frac{\log(fS_{ut}) - \log(S_e')}{6-3}$$

Endurance Limit Modifying Factors

- The fatigue strength of a machine element can be considerably different than that of a specimen made out of the same material used in laboratory fatigue tests, because, the fatigue strength is very sensitive to manufacturing and service conditions.
- Endurance limits of the specimen and the machine element of the same material are related as follows:

Endurance Limit Modifying Factors

$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$

- In this course, the following definitions of endurance limit modifying factors will be used.
- Note that for some factors, definitions are different from those given in the current edition of the text book.

Endurance Limit Modifying Factors

- k_a: Surface Factor
- k_b: Size Factor
- k_c: Reliability Factor
- k_d: Temperature Factor
- k_e: Stress Concentration Modifying Factor
- k_f: Miscellaneous factors

Surface Factor, k_a

- Surfaces of fatigue test specimens are polished in such a direction that the surface scratches do not start an early fatigue failure.

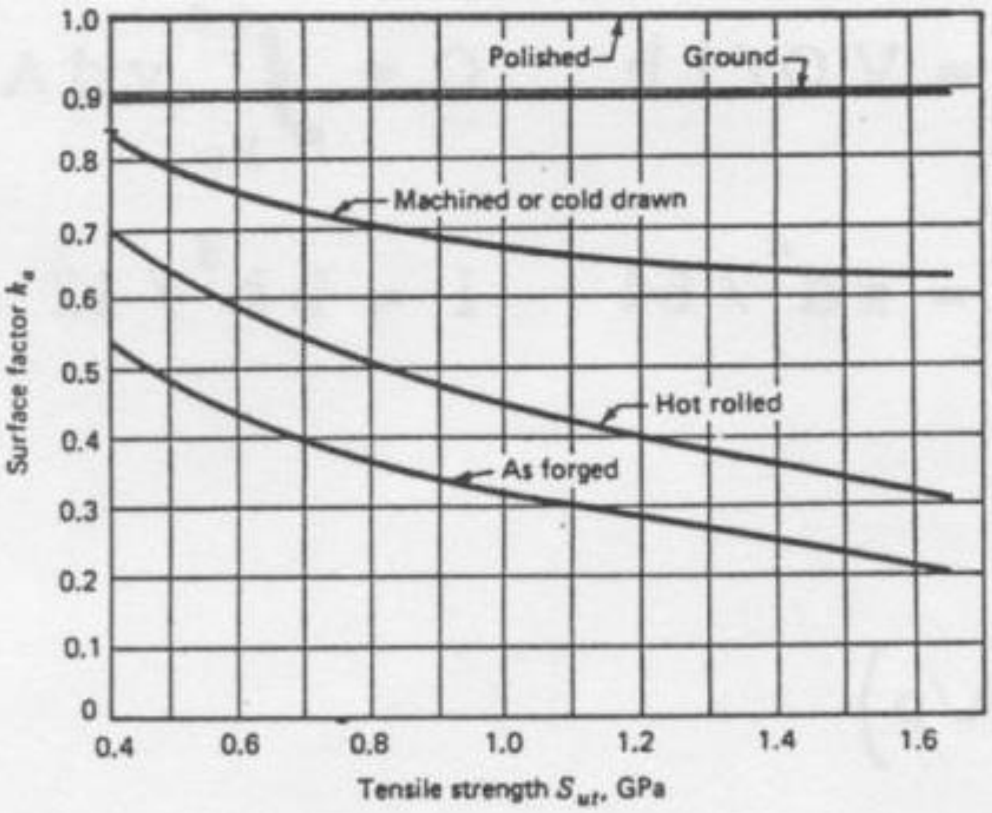


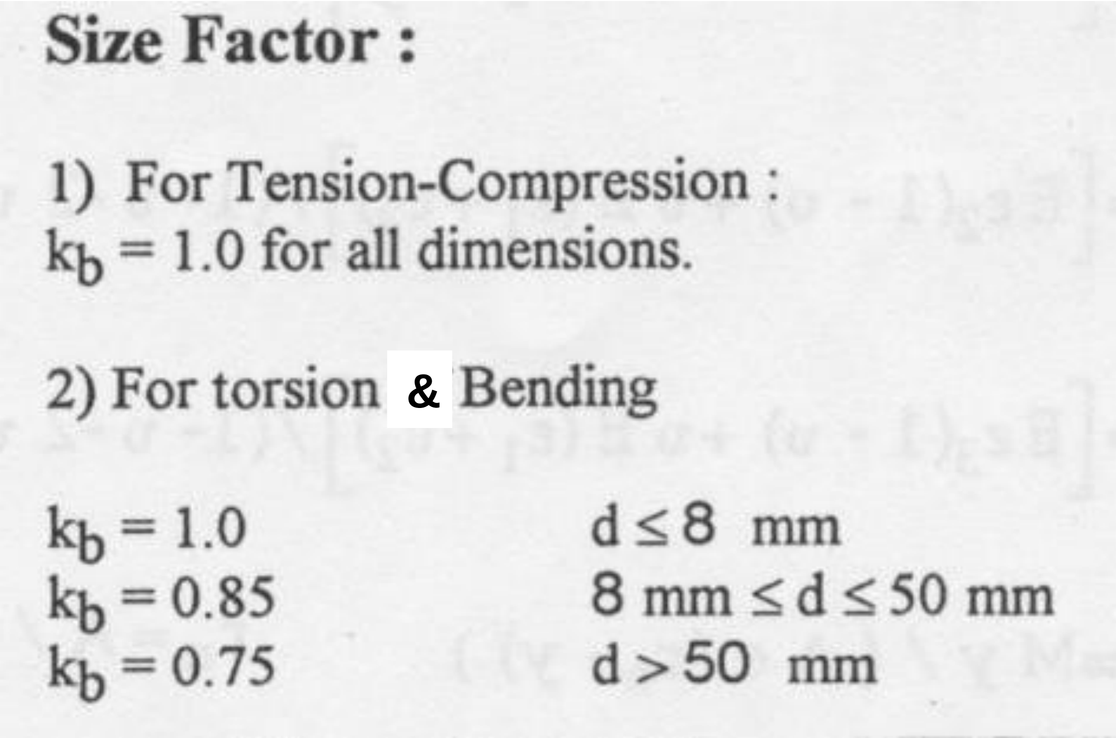
FIGURE 7-8 Surface-finish modification factors for steel. These are the k_a factors for use in Eq. (7-15).

Size Factor, k_b

- Endurance limits of machine elements with larger sizes and different cross sections are different (lower) from the endurance limit of the rotating beam test specimen.
- This effect due to dimension, shape and non-uniform stress distribution is called the size effect.
- Different theories are proposed to explain the reason of this effect. (See Shigley J.E., Mechanical Engineering Design, 1st metric edition)

Size Factor, k_b

- We adopt a simple approach in this course.



Size Factor :

1) For Tension-Compression :
$k_b = 1.0$ for all dimensions.

2) For torsion & Bending

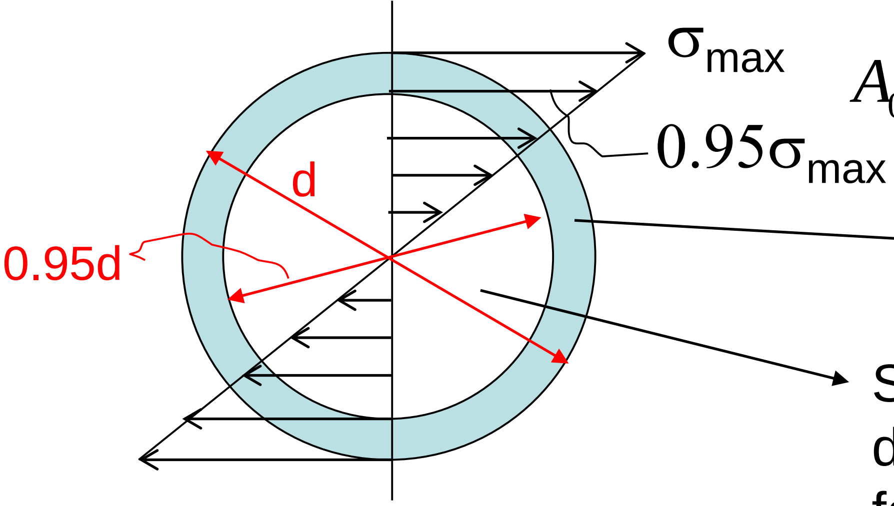
$k_b = 1.0$	$d \leq 8$ mm
$k_b = 0.85$	$8 \text{ mm} \leq d \leq 50 \text{ mm}$
$k_b = 0.75$	$d > 50$ mm

Size Factor, k_b

- In fatigue, the region of the machine element where stress level is below a certain value will not contribute to failure.
- Based on this fact, we use an effective dimension "d" for non-circular cross sections and non-rotating sections.
- Effective dimension "d" is found by equating the volume of material stressed at and above 95% of maximum stress to the same volume in rotating beam test specimen.
- Lengths cancel and we consider only areas.

Size Factor, k_b

Torsion and Bending (rotating beam)

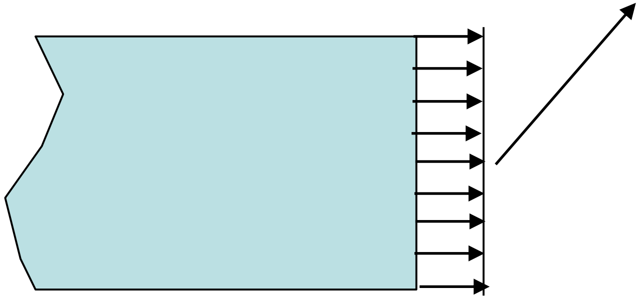


$$A_{0.95\sigma} = \frac{\pi}{4}\left[d^2 - (0.95d)^2\right] \approx 0.0766 d^2$$

Highly stressed region

Stresses are low and
do not contribute to
failure

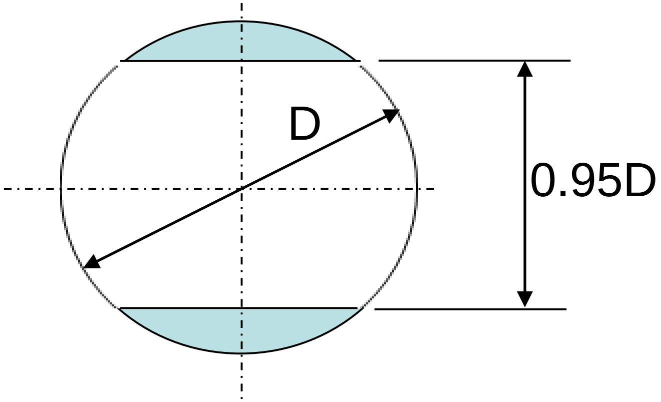
Axial loading



Cross section is uniformly stressed.
k_b=1

Size Factor, k_b

Non-Rotating Solid Round

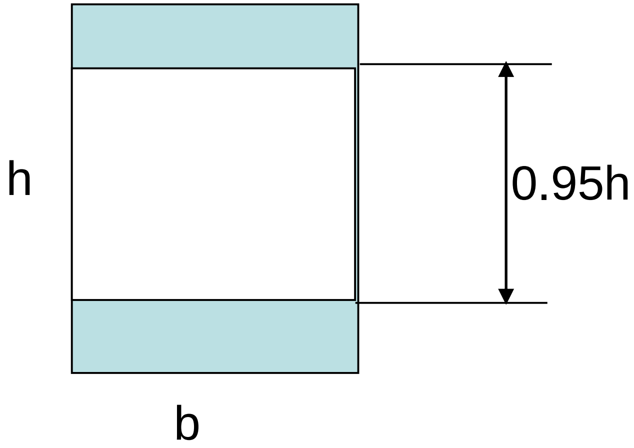


$$A_{0.95\sigma} \approx 0.0105D^2$$

$$0.0105D^2 = 0.0766d^2$$

$$d = 0.370D$$

Rectangular cross section



$$0.05bh = 0.0766d^2$$

$$d = \sqrt{\frac{0.05}{0.0766}hb}$$

$$d = 0.808\sqrt{hb}$$

Reliability Factor, k_c

- Fatigue tests do not give completely repeatable results and there is a scatter around a mean value. S-N diagram is drawn to pass through the mean value.
- For machine elements where higher reliability is required, reliability factor which is obtained by statistical methods is used.

Reliability Factor, k_c

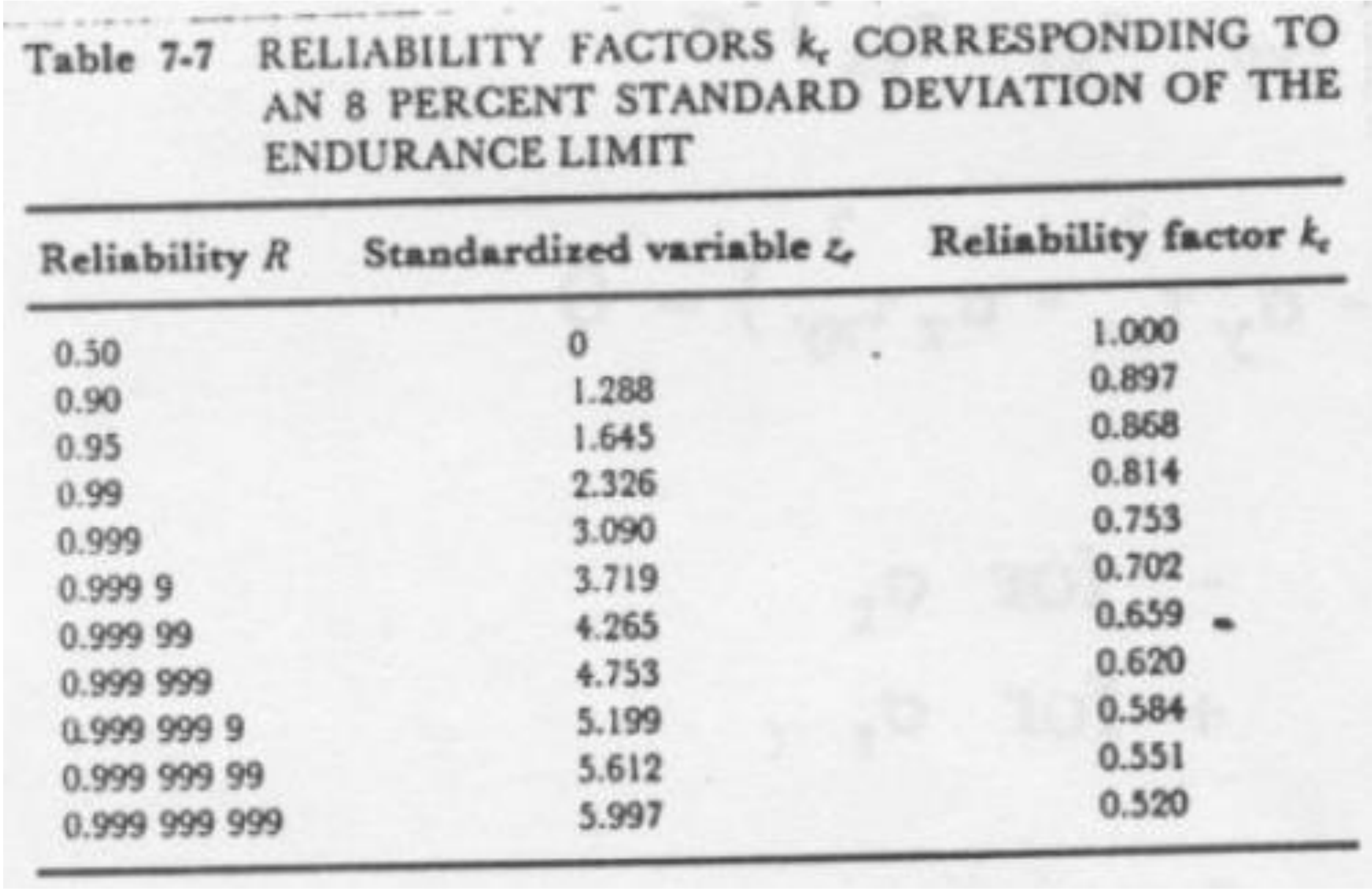
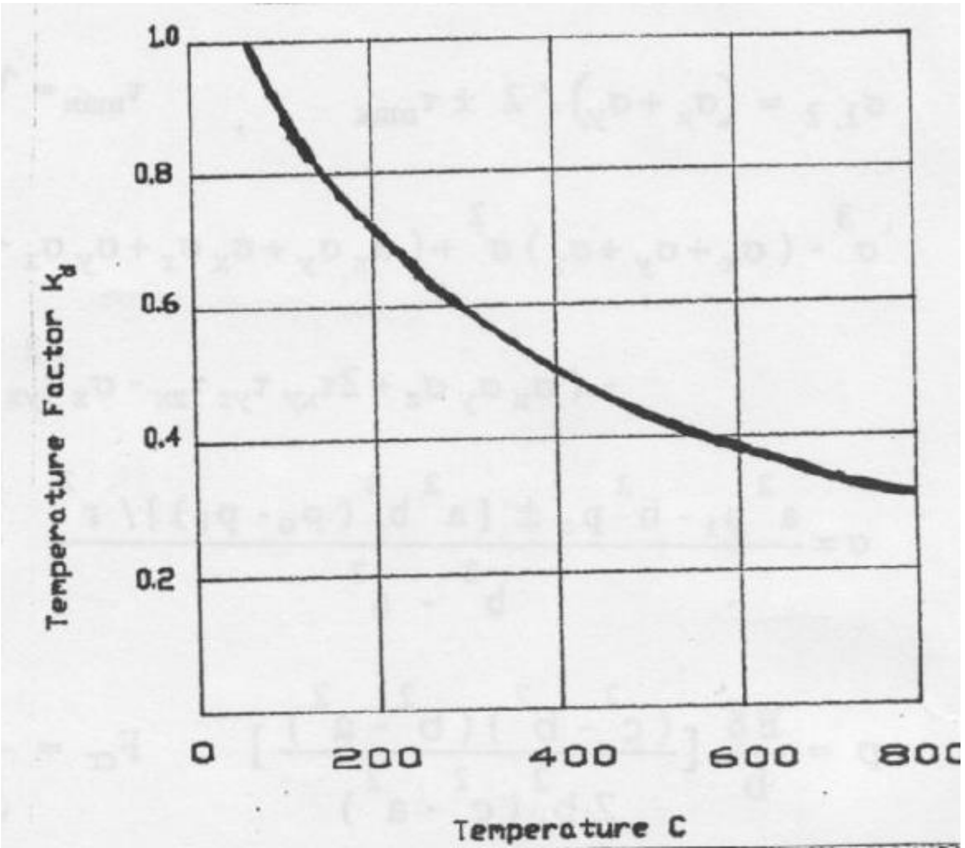


Table 7-7 RELIABILITY FACTORS k_r CORRESPONDING TO AN 8 PERCENT STANDARD DEVIATION OF THE ENDURANCE LIMIT

Reliability R	Standardized variable z_r	Reliability factor k_r
0.50	0	1.000
0.90	1.288	0.897
0.95	1.645	0.868
0.99	2.326	0.814
0.999	3.090	0.753
0.999 9	3.719	0.702
0.999 99	4.265	0.659
0.999 999	4.753	0.620
0.999 999 9	5.199	0.584
0.999 999 99	5.612	0.551
0.999 999 999	5.997	0.520

Temperature Factor, k_d

- Temperature is also a factor affecting endurance limit. Fatigue limit may not exist at high temperatures.



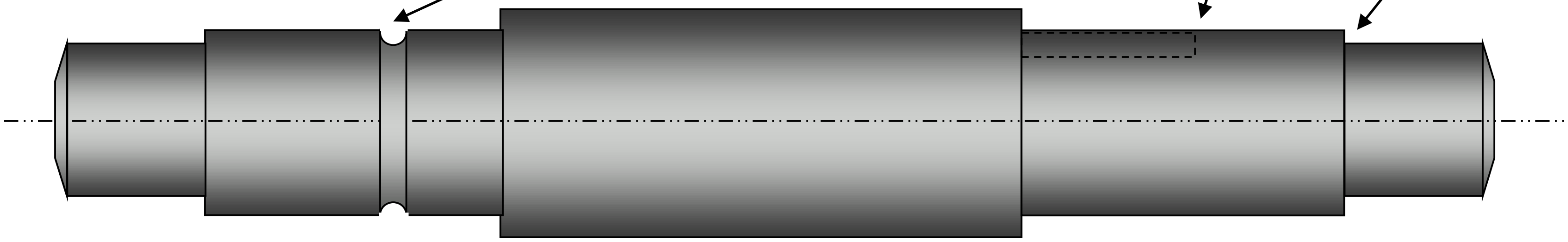
Fatigue Stress Concentration Modifying Factor, k_e

- Kt is the geometric or theoretical stress concentration factor since material properties are not considered.
- It was used only in brittle case for static loading.
- When there is fatigue loading, we always take stress concentration effects into account.
- We use fatigue stress concentration factor K_f.

Stress Concentration (Kt)

- Abrupt changes in the cross-section of the machine parts (discontinuities) may be necessary (holes, slots, shoulders etc.).
- Discontinuities alter the stress distribution in the vicinity of these discontinuities.
- Elementary stress equations are no longer valid in these regions. Such discontinuities are therefore called "stress raisers".
- These regions are called areas of "stress concentration." (highly localized effect).

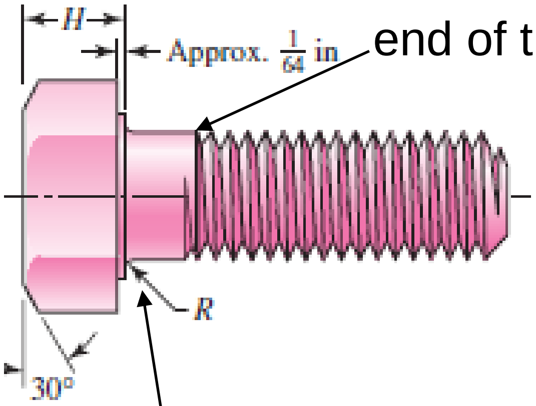
Examples of stress concentration



groove

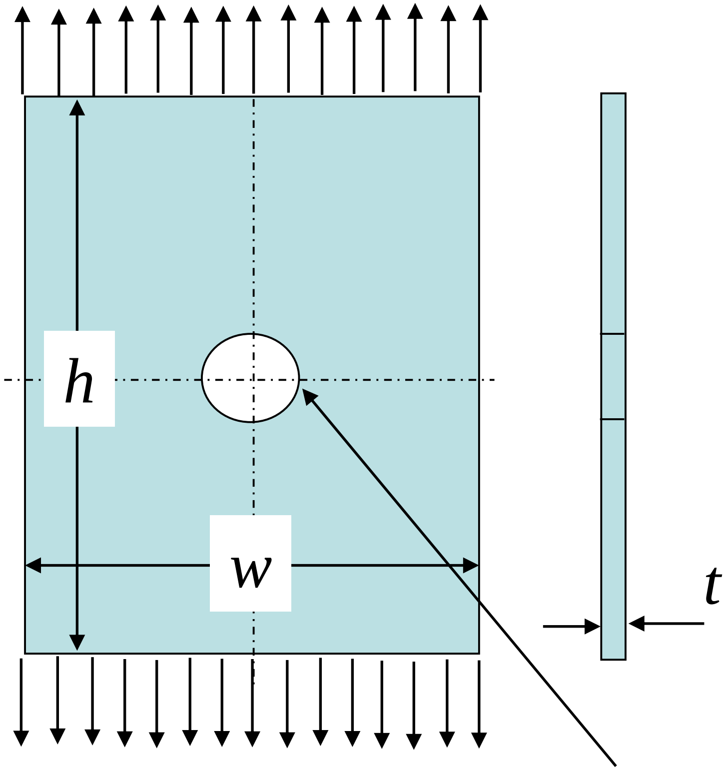
keyway (slot)

shoulder



end of thread

fillet at the bottom of the cap



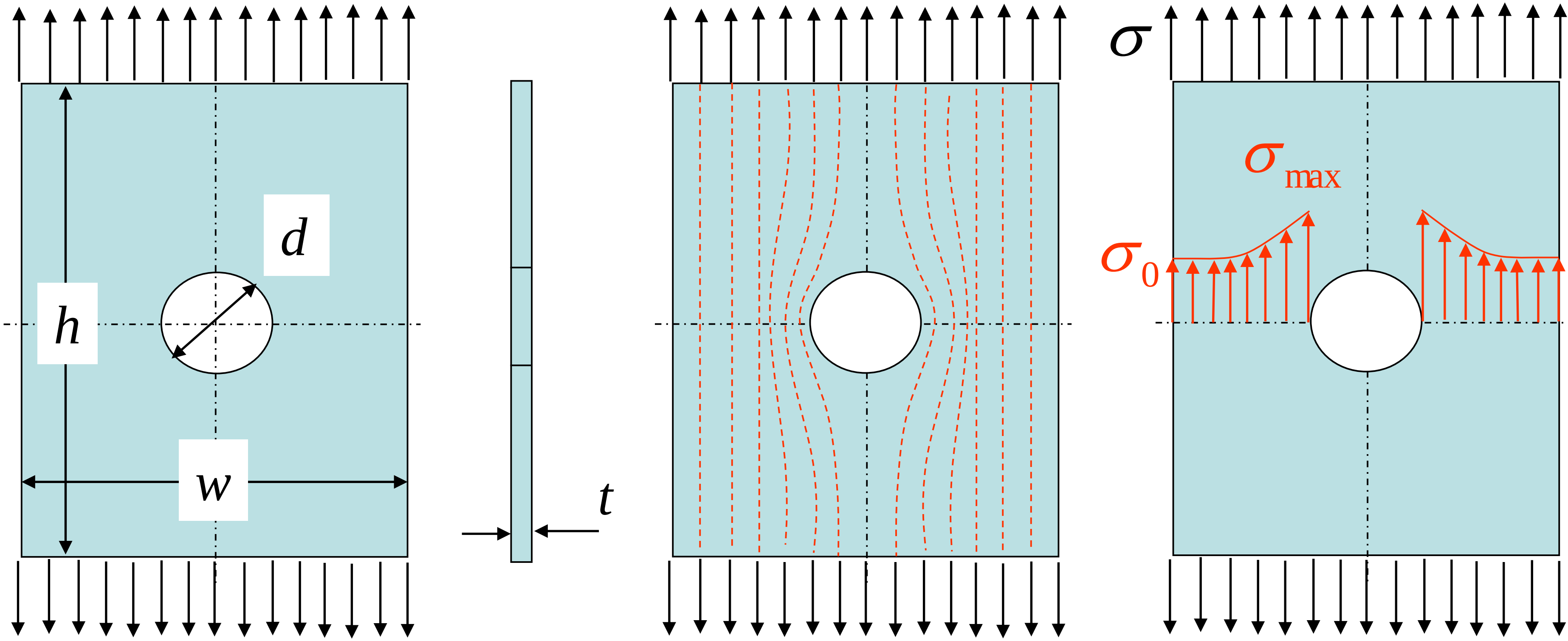
hole in the plate

Stress Concentration Factor 1

- A theoretical or geometric (i.e. does not depend on material) stress concentration factor is used to relate the actual maximum stress at the discontinuity to the nominal stress.

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \quad , \quad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

- σ_0 and τ_0 are calculated by elemantary stress equations and by using either the net or the gross (un-notched) cross-sectional area.
- K_{ts} and K_t values for different geometries are presented in tables and charts.



nominal stress

$$\sigma_0 = \frac{F}{(w-d)t}$$

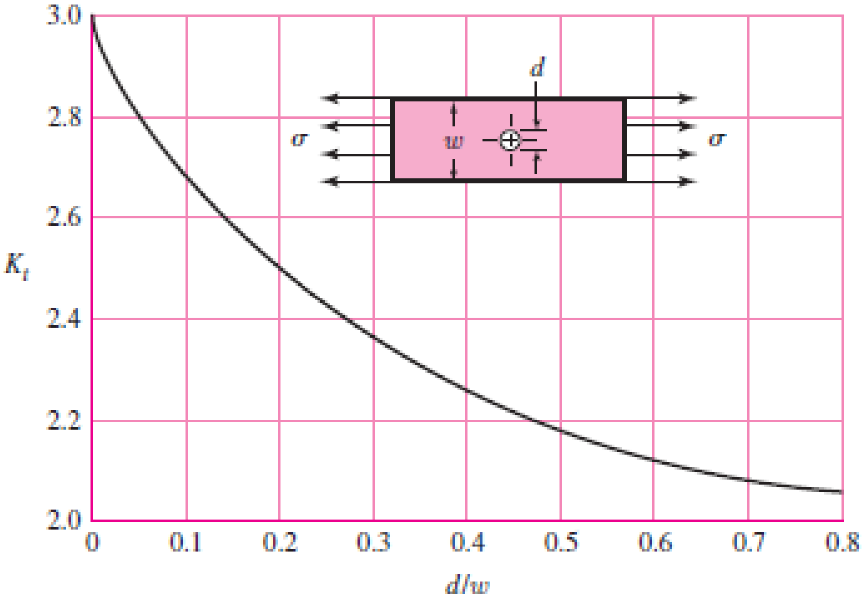
$$\sigma_{max} = K_t \sigma_0$$

$$= K_t \frac{F}{(w-d)t}$$

Figure 3–29

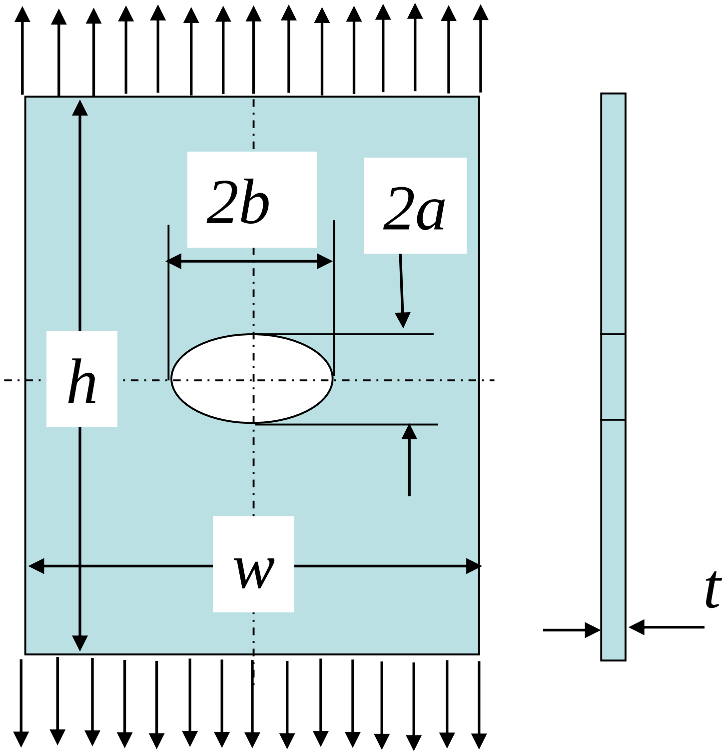
Thin plate in tension or simple compression with a transverse central hole. The net tensile force is $F = \sigma w t$, where t is the thickness of the plate. The nominal stress is given by

$$\sigma_0 = \frac{F}{(w-d)t} = \frac{w}{(w-d)}\sigma$$



Stress Concentration Factor 2

- Stress concentration factors are determined by using analytical, experimental or numerical methods.
- In general, as the change in the cross-sectional area is more gradual (i.e. less abrupt), stress concentration becomes lower.
- Hence, sometimes removing material reduces stress concentration.



$$K_t = 1 + \frac{2b}{a} \quad w \gg a, b \quad h \gg a, b$$

a/b	K_t
2	2
1	3
1/2	5

Notch Sensitivity 1

- K_t (K_{ts}) depends only on the geometry. However it has been found that some materials are not fully sensitive to the presence of notches.
- So under certain conditions a reduced value of K_t may be used. Instead of $\sigma_{max}=K_t\sigma_0$ one can use $\sigma_{max}=K_f\sigma_0$.
- K_f depends on material as well as geometry and it is commonly called fatigue stress concentration factor because it is used for fatigue (cyclic) loading.

Notch Sensitivity 2

$$K_f = \frac{\text{max. stress in notched specimen}}{\text{stress in notch-free specimen}} \qquad q = \frac{K_f - 1}{K_t - 1}$$

- q: notch sensitivity
- $q=0$, $K_f=1$ no sensitivity to notches
- $q=1$, $K_f=K_t$ full sensitivity to notches
- In fatigue problems, we first find K_t for the given geometry, then find q for the specified material by using the given charts and determine K_f.

$$K_f = 1 + q(K_t - 1)$$

Notch Sensitivity 3

- If no reliable data exists on q, we can use K_t.
- For <u>ductile materials under static loading</u>, local yielding will relieve stress concentration and K_t is usually not used.
- <u>For brittle materials under static loading</u> K_t must be used, (applied to nominal stress).
- For cast iron, although it is brittle, K_t may not be used.(There are many internal imperfections causing stress concentration and reported strength takes them into account, so additional features have a marginal effect.)

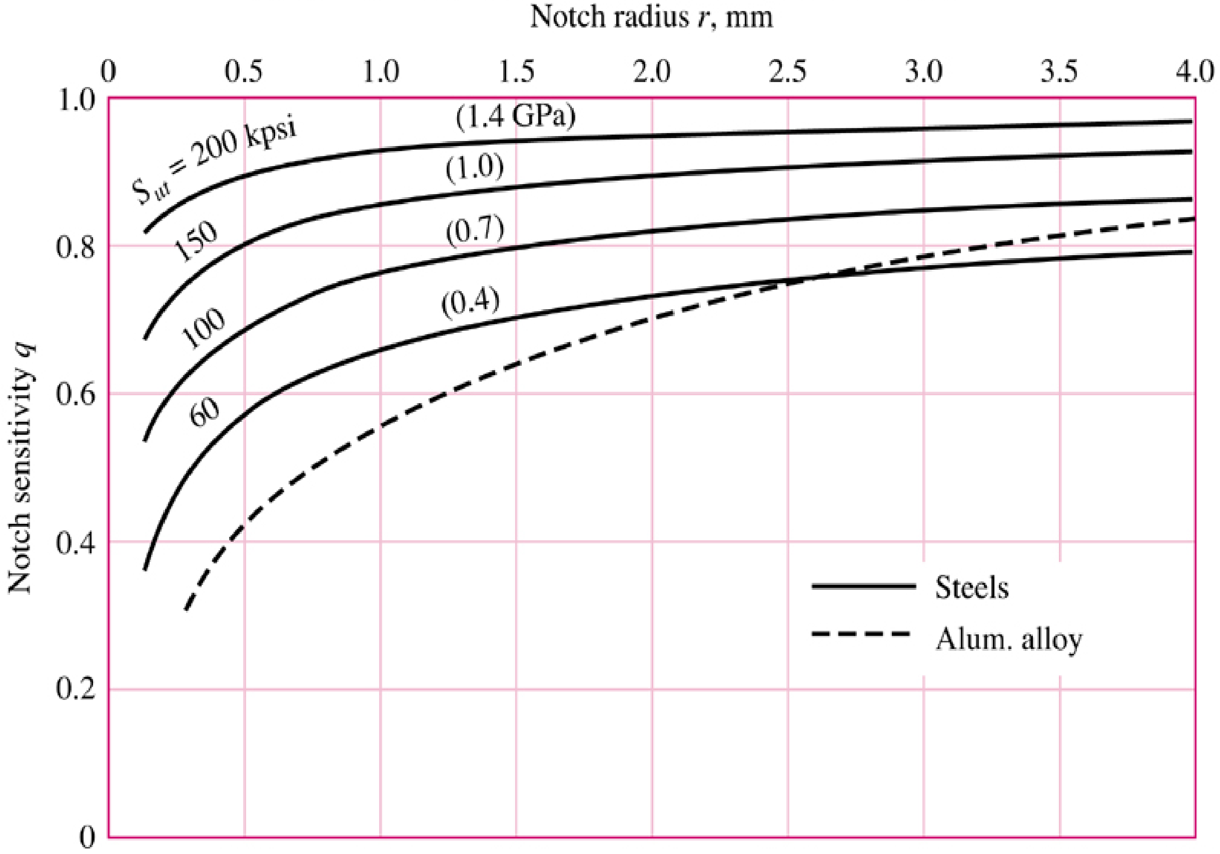


FIGURE 7-13 Notch-sensitivity charts for steels and 2024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii use the values q corresponding to $\mathrm{r = 4mm}$

(*By permission from George Sines and J. L. Waisman (eds.), Metal Fatigue, McGraw-Hill New York, 1959, pp. 296-298*)

Fatigue Stress Concentration Modifying Factor, k_e

$$K_f = 1 + q(K_t - 1)$$

- q is the notch sensitivity, and it is a property of material,notch geometry and loading.
- In general $0<q<1$. $q=0$ means material is insensitive to notches and $q=1$ means material is fully sensitive to notches.
- Instead of multiplying stress with K_f, we can divide strength by K_f.

$$k_e = \frac{1}{K_f}$$

Fatigue Stress Concentration Modifying Factor, k_e

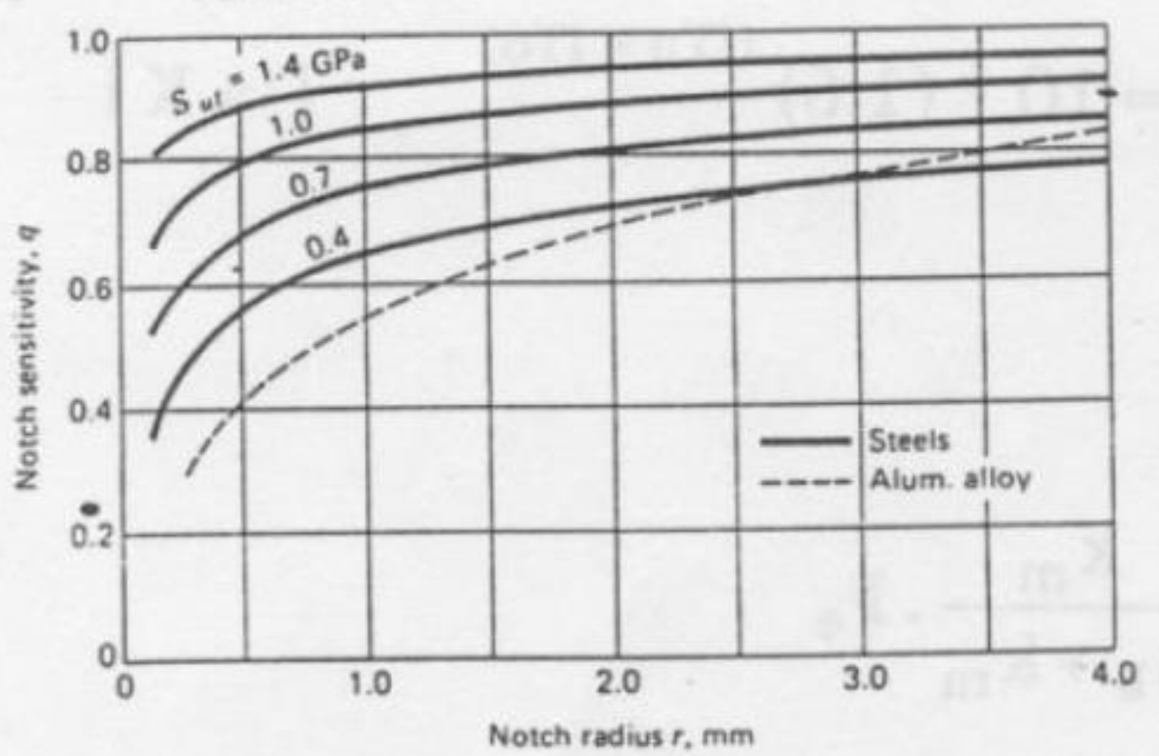


FIGURE 7-13 Notch-sensitivity charts for steels and 2024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii use the values of q corresponding to $r = 4$ mm. (*By permission from George Sines and J. L. Waisman (eds.), Metal Fatigue, McGraw-Hill, New York, 1959, pp. 296–298.*)

Fatigue Stress Concentration Modifying Factor, k_e

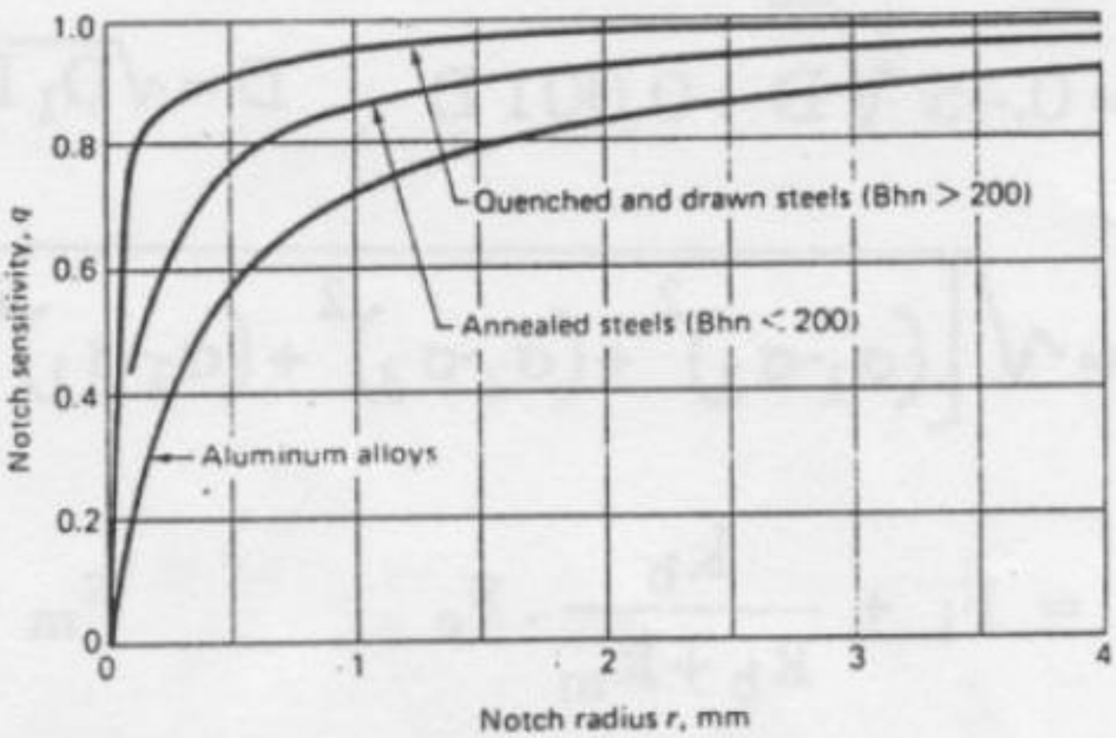


FIGURE 7-14 Notch-sensitivity curves for materials in reversed torsion. For larger notch radii use the values of q corresponding to $r = 4$ mm.

Other Factors, k_f

- Though the factor k_f is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for because actual values of k_f are not available.
- Some factors which may effect endurance limit are as follows:

Other Factors, k_f

- Residual Stress: Fatigue failures are tensile failures, so tensile residual stresses must be avoided.(Resultant stress may become too large.) On the other hand, compressive residual stresses improve fatigue resistance. Shot peening, cold working, case hardening improve fatigue strength.
- Corrosion:Generally, corrosive atmosphere will lower the endurance limit.

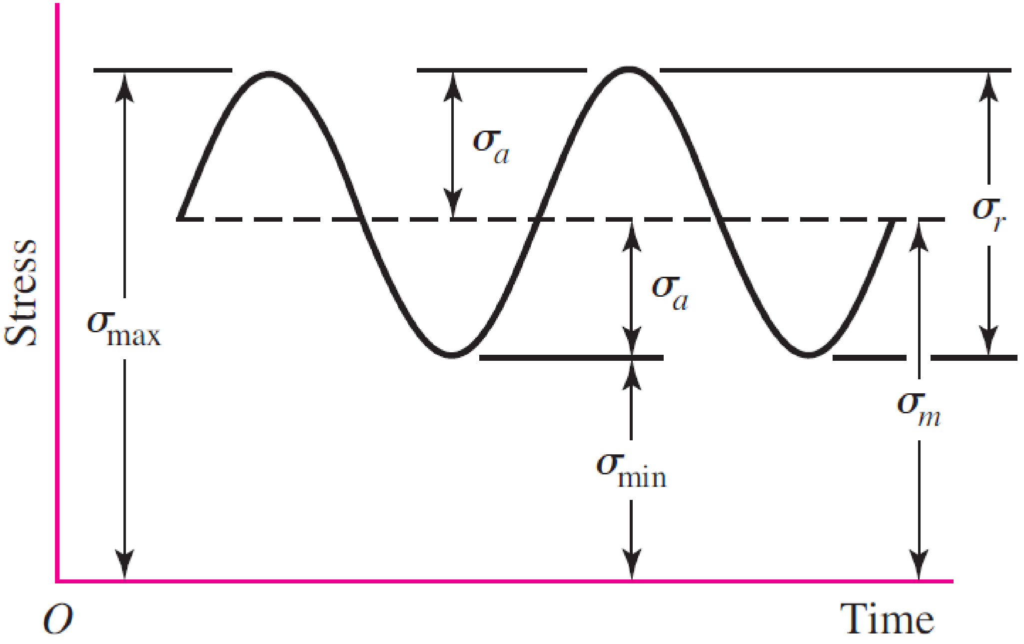
Other Factors, k_f

- Plating: Metallic coatings such as chromium, nickel or cadmium reduce the endurance limit by as much as 50%. Zinc plating is harmless.
- Cyclic frequency:Under normal conditions fatigue failure is independent of frequency. But when there is corrosion and/or high temperature, cyclic rate becomes important.
- Frettage corrosion: Microscopic relative motion of tightly fitting parts or structures such as bolted joints, wheels, hubs on shaft etc. causes fretting.

Fully Reversed Stress

- Once the strength is determined, (S_e or S_f for a given N) it is compared with σ_{max}.
- If $\sigma_{max}<S_e$ we have infinite life. Otherwise S-N diagram should be used.
- Factor of safety or dimensions of the part are determined.
- Note that S_e can change from one section to another on the same part due to different stress concentrations, surface conditions, etc.

Fluctuating Stress



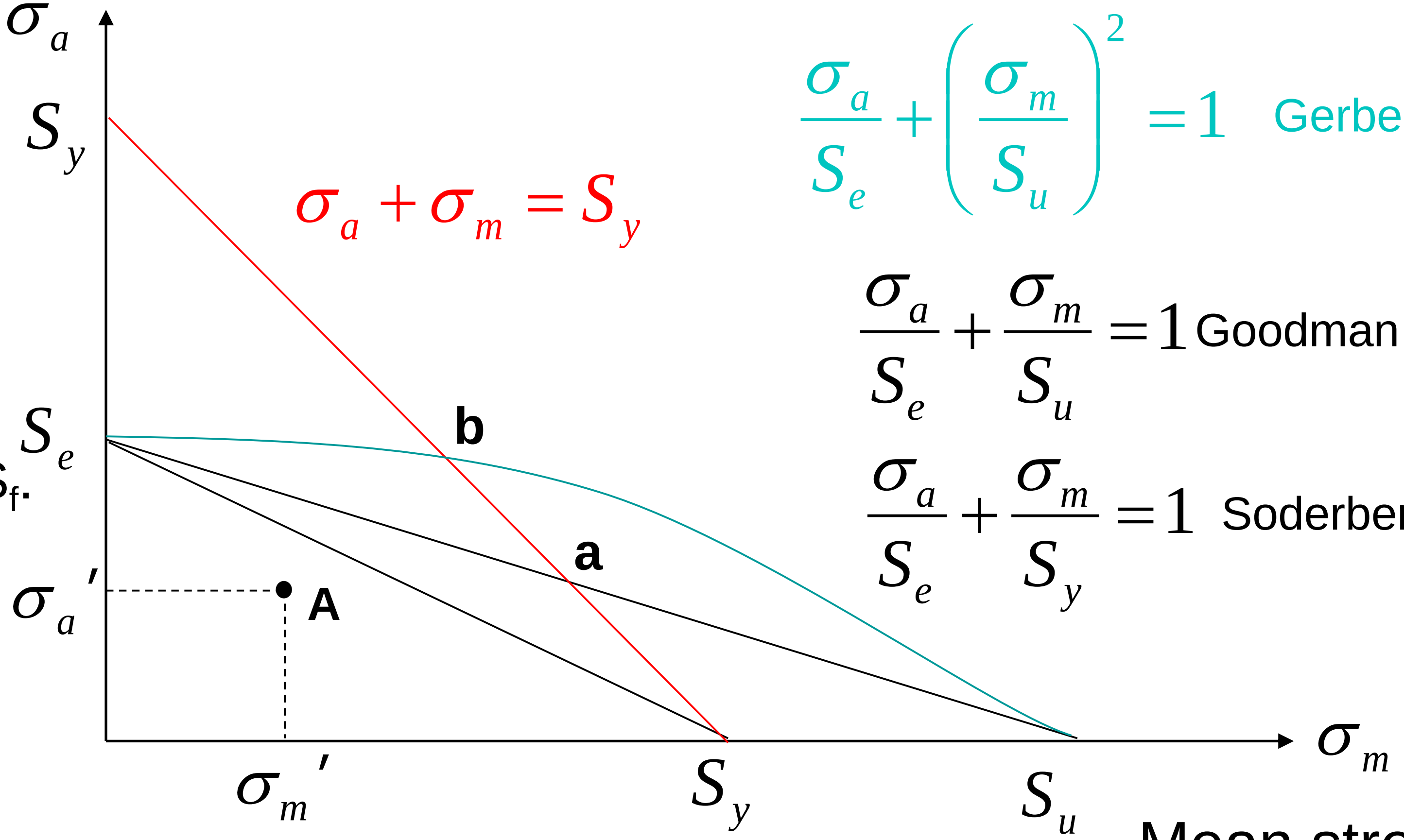
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

- As the mean stress increases, generally permitted alternating stress to cause fatigue failure decreases.
- There are a number of hypotheses proposed to take the effect of mean stress into account.

Fluctuating Stress

Alternating stress σ_a



$$\sigma_a + \sigma_m = S_y$$

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1$$ Gerber line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$$ Goodman line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$$ Soderberg line

For finite life we can use S_f.

Mean stress

Fluctuating Stress

- Safety Factor at point A for infinite life

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_y} = \frac{1}{n}$$

According to Soderberg Line

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_u} = \frac{1}{n}$$

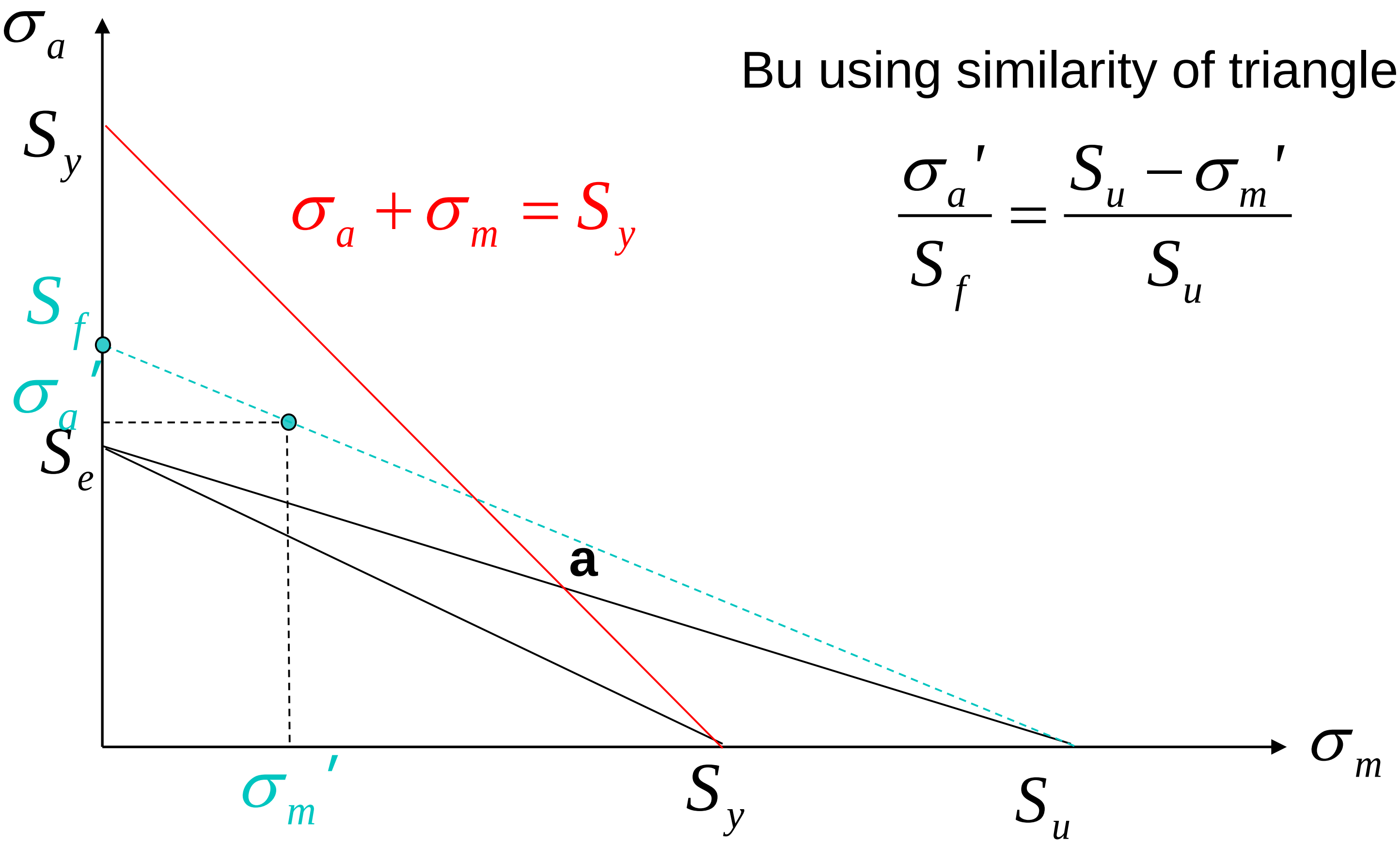
According to Goodman Line
(intersection of load and goodman line to the left of a)

$$\frac{n\sigma_a'}{S_e} + \left(\frac{n\sigma_m'}{S_u}\right)^2 = 1$$

According to Gerber Line
(intersection of load and goodman line to the left of b)

Fluctuating Stress

- Note that Goodman or Soderberg approach can be used to estimate finite life under fluctuating stress.

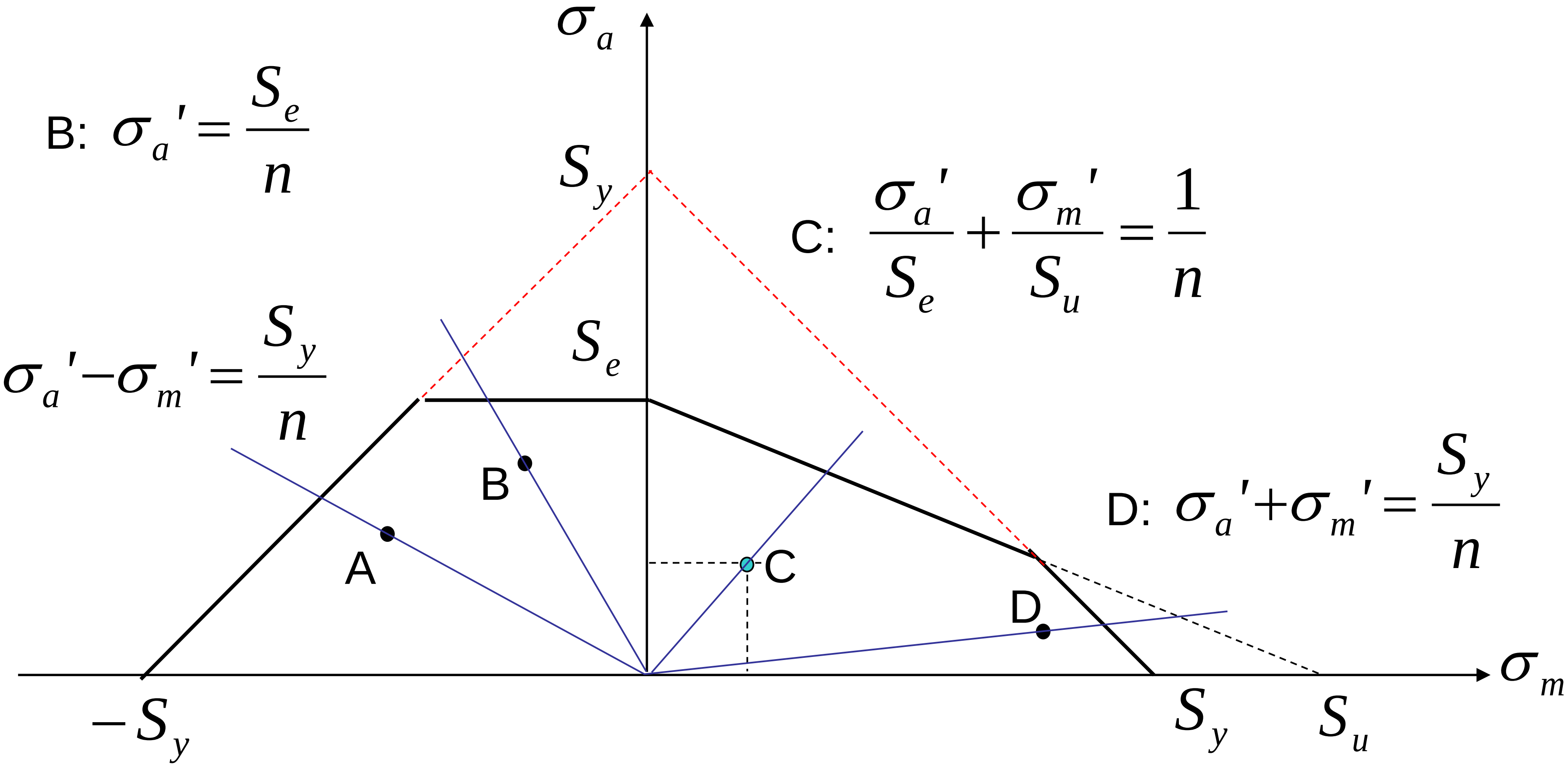


Bu using similarity of triangles

$$\frac{\sigma_a'}{S_f} = \frac{S_u - \sigma_m'}{S_u}$$

Fluctuating Stress

Extension of Modified Goodman Approach to Compressive Mean Stresses:



B: $\sigma_a' = \frac{S_e}{n}$

C: $\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_u} = \frac{1}{n}$

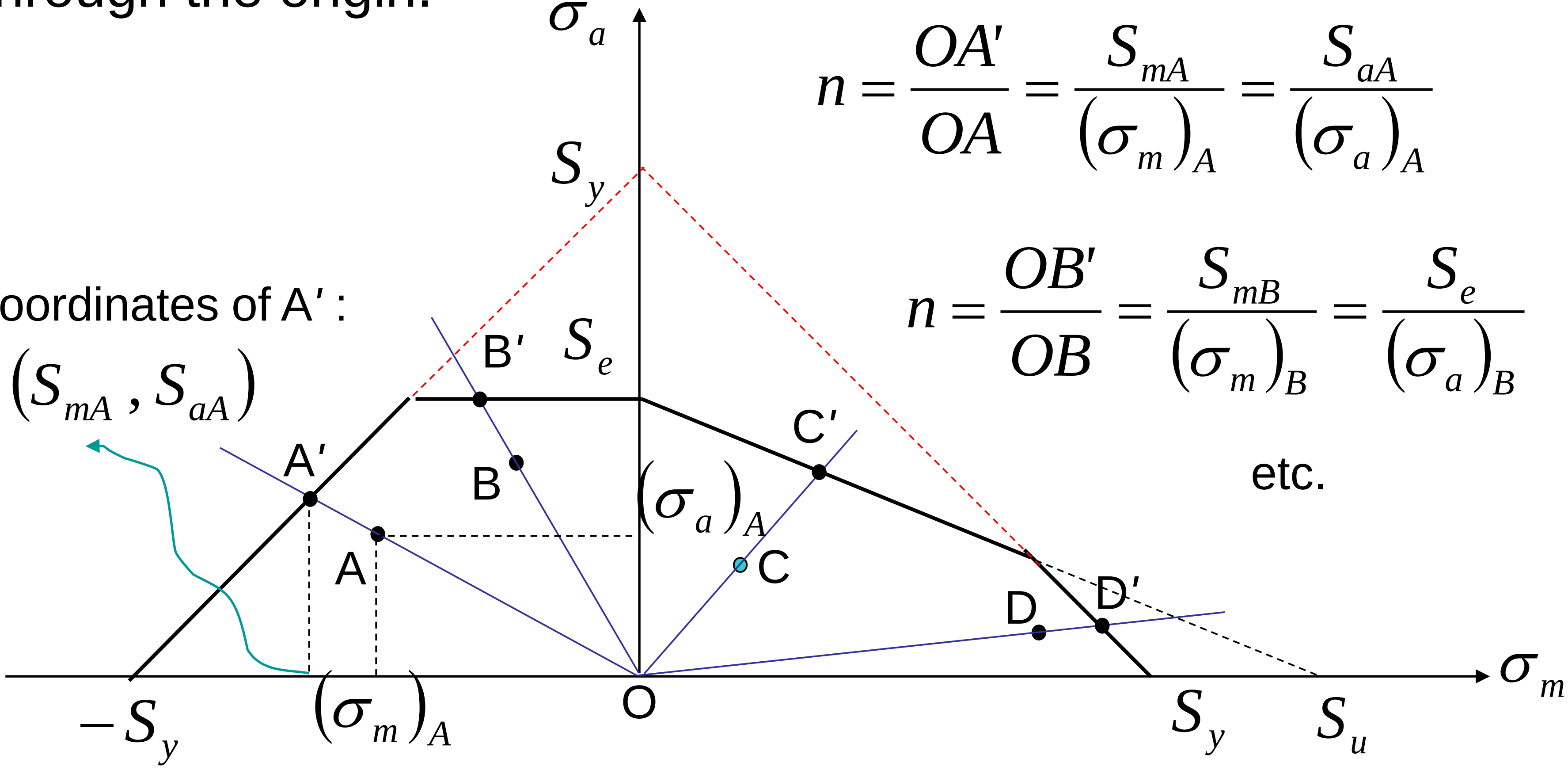
A: $\sigma_a' - \sigma_m' = \frac{S_y}{n}$

D: $\sigma_a' + \sigma_m' = \frac{S_y}{n}$

Fluctuating Stress

Let the mean and alternating stresses are assumed to be proportional to the load.

Then the load lines are assumed to be straight and passing through the origin.



$$n = \frac{OA'}{OA} = \frac{S_{mA}}{(\sigma_m)_A} = \frac{S_{aA}}{(\sigma_a)_A}$$

$$n = \frac{OB'}{OB} = \frac{S_{mB}}{(\sigma_m)_B} = \frac{S_e}{(\sigma_a)_B}$$

etc.

Coordinates of A′ :

(S_{mA}, S_{aA})

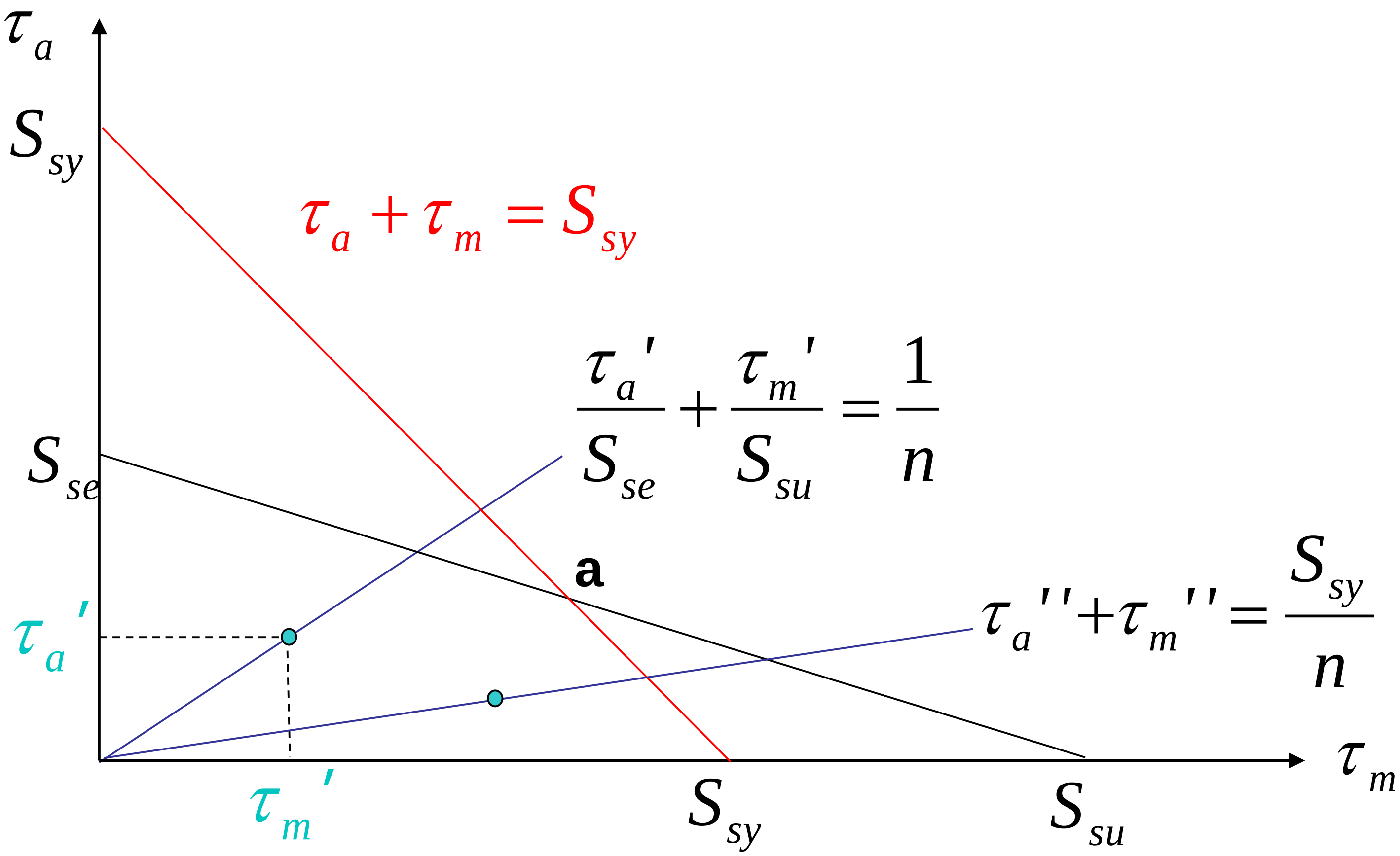
Pure Torsion

- Recall that in static design criteria we used S_{sy}=0.5S_y (MSST) or S_{sy}=0.577S_y (DET).
- Interestingly, experiments indicate that if Se is known, S_{se}=0.5S_e or S_{se}=0.577S_e give reasonably good results.
- Then for fully reversed torsion,

$$n = \frac{S_{se}}{\tau_a}$$

Pure Torsion

- For fluctuating torsion we can draw the following diagram, by taking S_{su}=0.67S_u.



$$\tau_a + \tau_m = S_{sy}$$

$$\frac{\tau_a'}{S_{se}} + \frac{\tau_m'}{S_{su}} = \frac{1}{n}$$

$$\tau_a'' + \tau_m'' = \frac{S_{sy}}{n}$$