Failure theories for materials with different tensile and compressive strengths

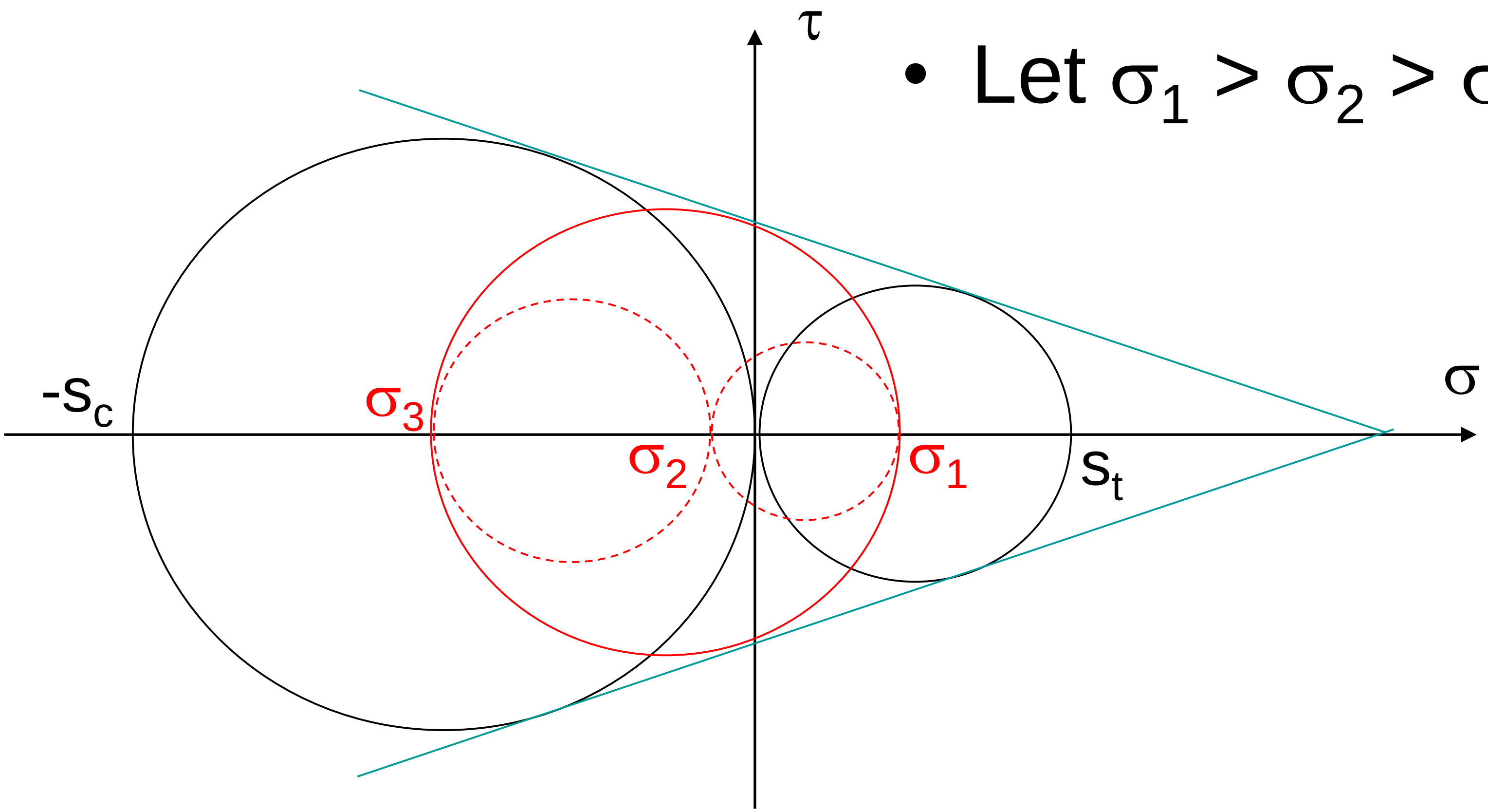
Overview

- Some materials have different tensile and compressive strengths
  - For magnesium $S_{ty} \approx 2S_{cy}$
  - For gray cast iron $3\sim4S_{ut} \approx S_{uc}$
- The following theories will be covered
  - Coulomb-Mohr Theory
  - Modified Mohr Theory

Coulomb Mohr Theory

- According to Coulomb Mohr Theory, the onset of failure is predicted when the 3D Mohr circle of a given stress state becomes tangent to the common tangent line of two Mohr circles, which are
  - Mohr circle for uniaxial tension at failure
  - Mohr circle for uniaxial compression at failure

Coulomb Mohr Theory

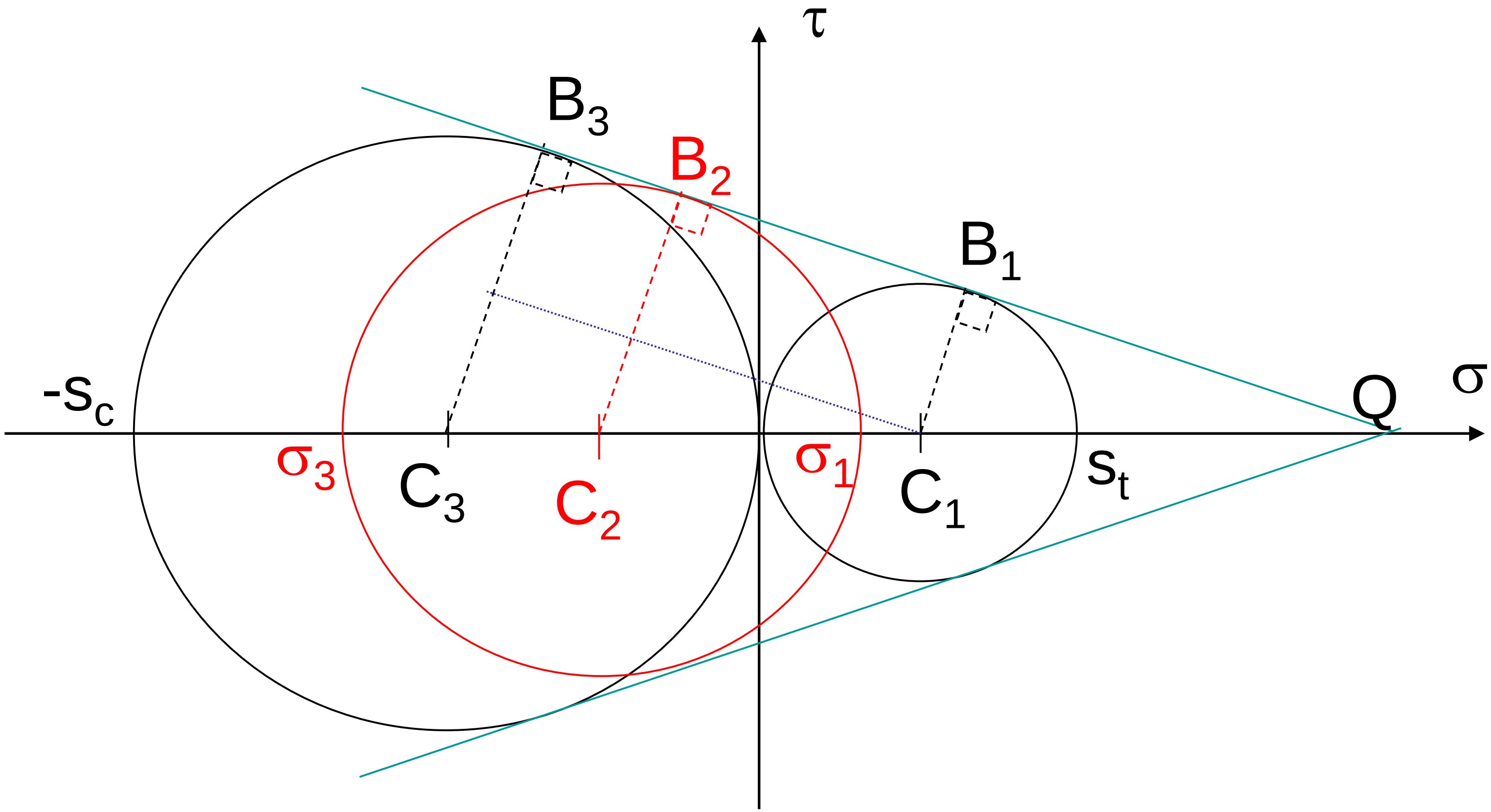


- Let $\sigma_1 > \sigma_2 > \sigma_3$

- Tangent lines define the failure envelope
- If the 3D Mohr circle is inside these lines, there is no failure
- When 3D Mohr circle becomes tangent as shown, failure begins

Coulomb Mohr Theory

- Tangency condition is $$\frac{B_2C_2 - B_1C_1}{B_3C_3 - B_1C_1} = \frac{QC_2 - QC_1}{QC_3 - QC_1}$$



Coulomb Mohr Theory

$$\frac{B_2C_2 - B_1C_1}{B_3C_3 - B_1C_1} = \frac{QC_2 - QC_1}{QC_3 - QC_1} = \frac{|C_1C_2|}{|C_1C_3|}$$

- Coordinates of C_1, C_2, C_3:

$$C_1(\frac{S_t}{2},0) \qquad C_2(\frac{\sigma_1+\sigma_3}{2},0) \qquad C_3(-\frac{S_c}{2},0)$$

- Lengths BC_1, BC_2, BC_3:

$$|BC_1| = \frac{S_t}{2} \qquad |BC_2| = \frac{\sigma_1-\sigma_3}{2} \qquad |BC_3| = \frac{S_c}{2}$$

$$\frac{\frac{\sigma_1-\sigma_3}{2} - \frac{S_t}{2}}{\frac{S_c}{2} - \frac{S_t}{2}} = \frac{\frac{S_t}{2} - \frac{\sigma_1+\sigma_3}{2}}{\frac{S_t}{2} - \left(-\frac{S_c}{2}\right)}$$

(simplifying)

$$\boxed{\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1}$$

Coulomb Mohr Theory

- We can estimate the strength under pure shear by using the equation derived.

Substitute $\sigma_1 = -\sigma_3 = \tau_{\max}$ $\longrightarrow$ $\dfrac{\sigma_1}{S_t} - \dfrac{\sigma_3}{S_c} = 1$

$$\frac{\tau_{\max}}{S_t} - \frac{-\tau_{\max}}{S_c} = 1 \longrightarrow \tau_{\max} = S_s = \frac{S_t S_c}{S_t + S_c}$$

- In design, we can introduce a factor of safety, n, and we have

$$\boxed{\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}}$$

Coulomb Mohr Theory

- For a biaxial case let, σ_A and σ_B be the non-zero principal stresses.
- If both σ_A and σ_B are positive (σ_3=0, is the smallest stress) then Max(σ_A, σ_B)$\geq S_t$ means failure .
- If both σ_A and σ_B are negative (σ_1=0, is the largest stress) then Min(σ_A, σ_B)$\leq -S_c$ means failure .

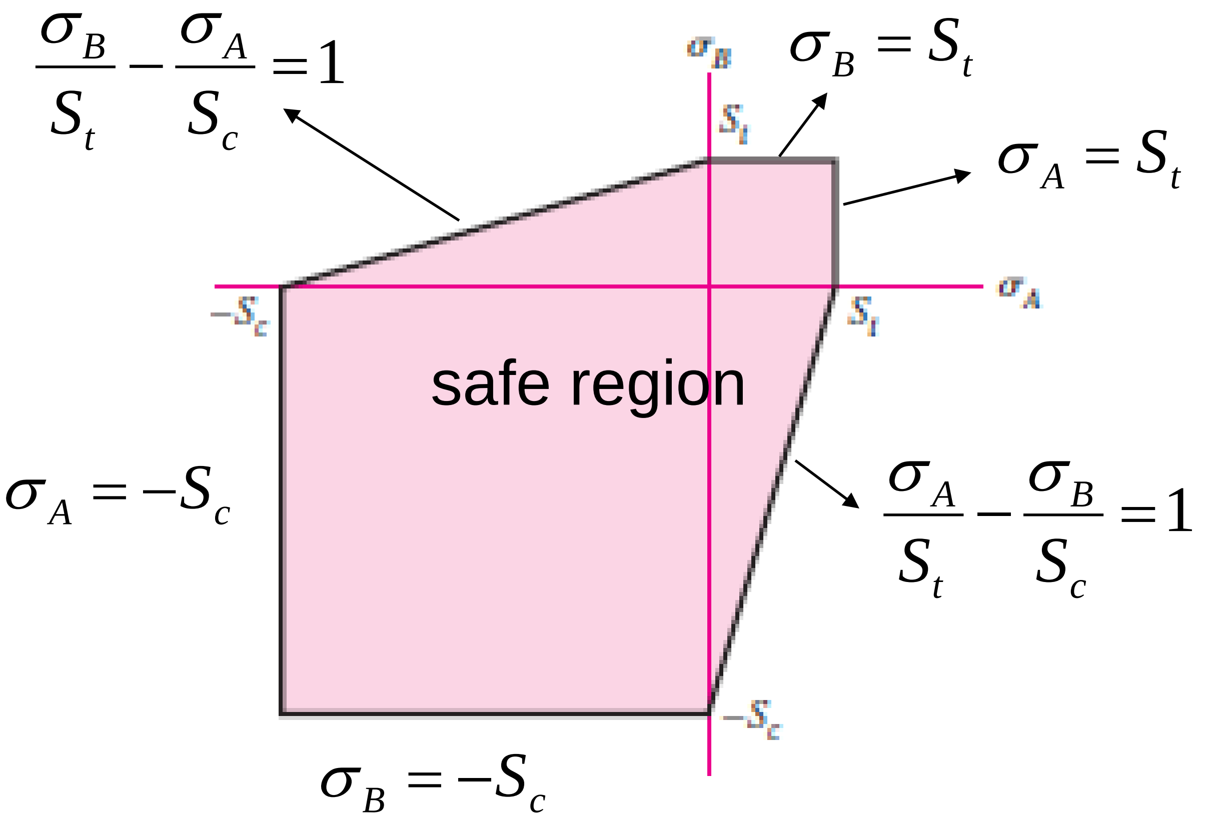
Coulomb Mohr Theory

- If σ_A and σ_B have opposite signs then σ_3=0 and either

$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1$$ or $$\frac{\sigma_B}{S_t} - \frac{\sigma_A}{S_c} \geq 1$$ means failure.

- These inequalities define the safe region for biaxial case as shown below.

Coulomb Mohr Theory

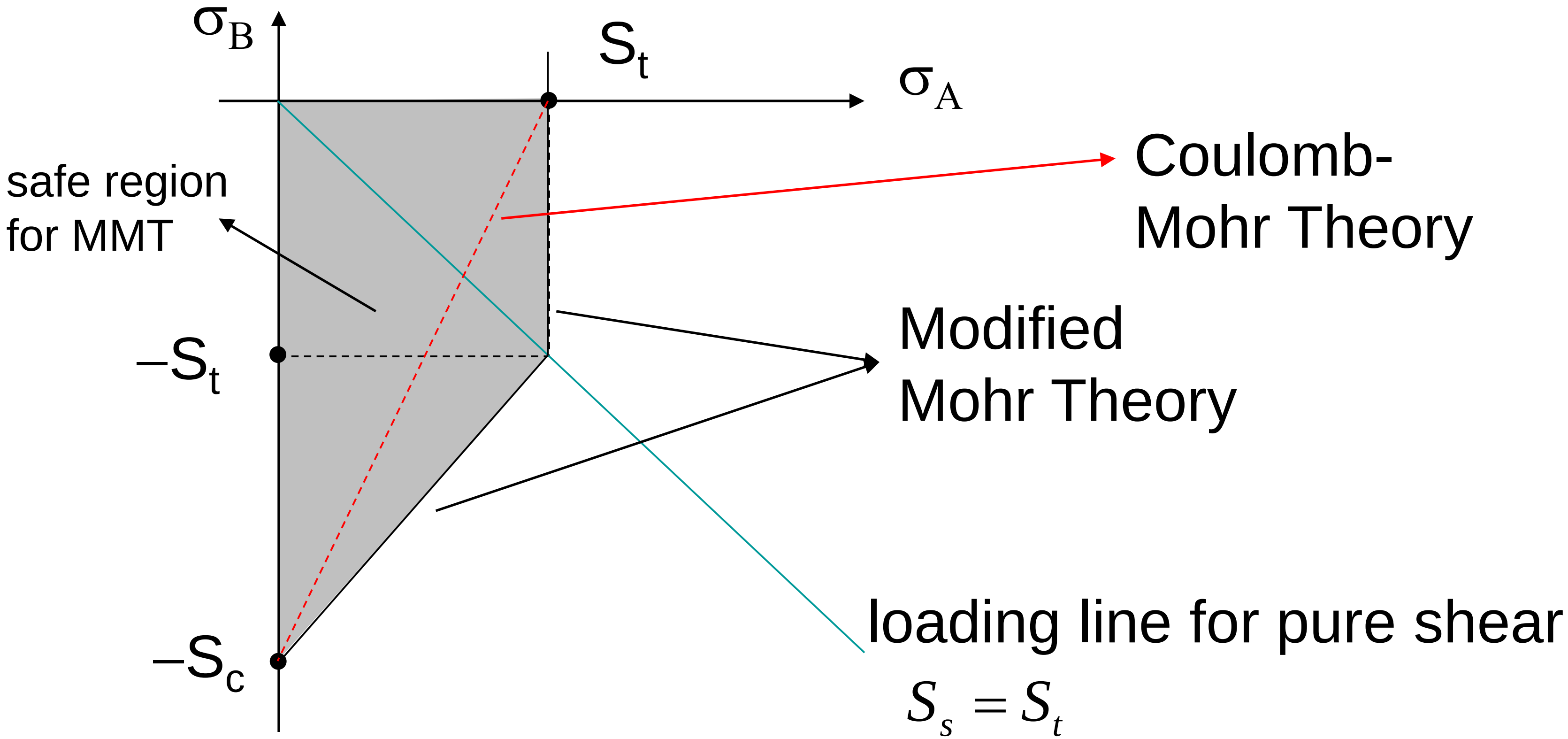


Modified Mohr Theory

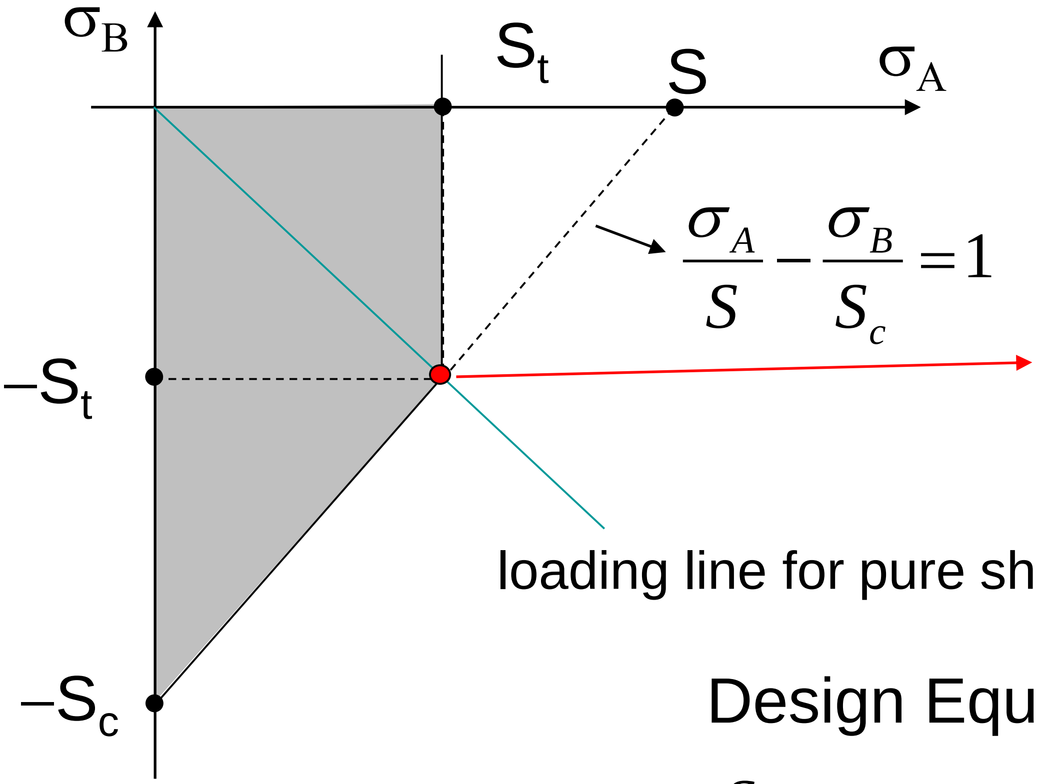
- Coulomb-Mohr theory is modified for the case when non zero principal stresses have opposite signs, so that better agreement with experiments is obtained.
- Modification is based on the observation that for many brittle materials, ultimate shear strength is roughly the same as ultimate tensile strength.

Modified Mohr Theory

- Without losing generality, let $\sigma_A > 0$ and $\sigma_B < 0$ (So we concentrate on the 4th quadrant in stress space).



Modified Mohr Theory



$$\frac{\sigma_A}{S} - \frac{\sigma_B}{S_c} = 1$$

point $(S_t,-S_t)$ satifies this equation so,

$$S = \frac{S_c S_t}{S_c - S_t}$$

Design Equations

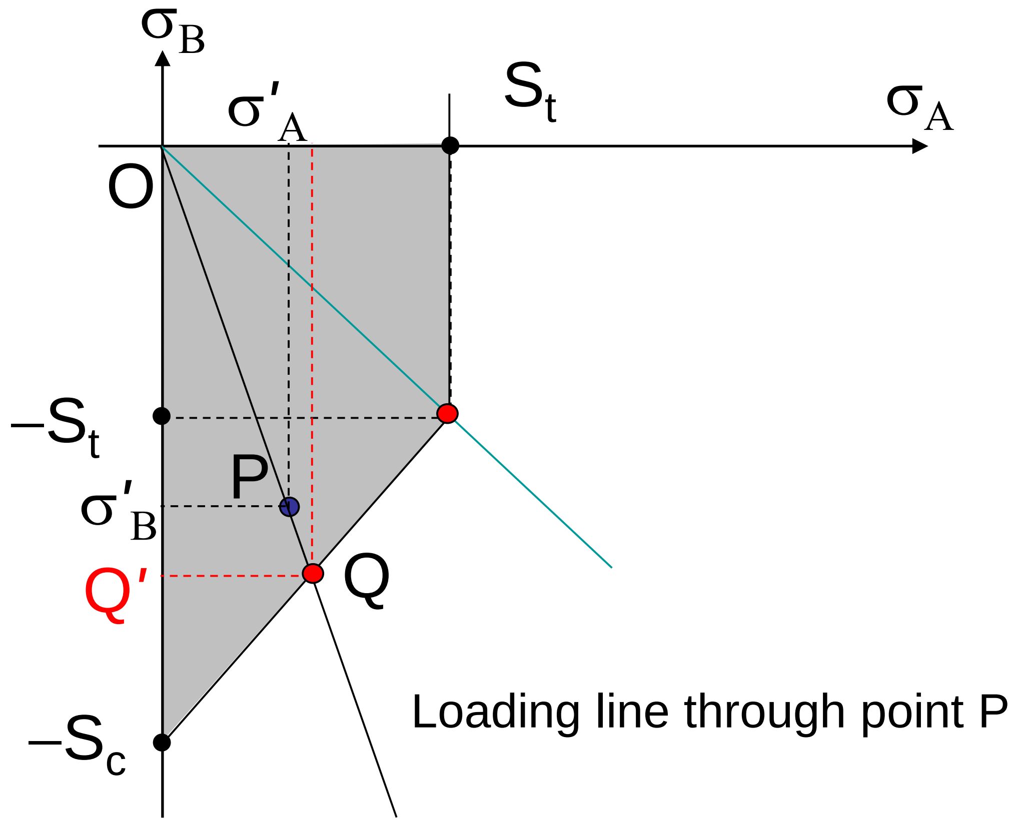
if $|\sigma_A/\sigma_B| \geq 1$ then $\sigma_A = \frac{S_t}{n}$ point (σ_A,σ_B) is on or above loading line for pure shear

if $|\sigma_A/\sigma_B| < 1$ then $\frac{\sigma_A}{S} - \frac{\sigma_B}{S_c} = \frac{1}{n}$ point (σ_A,σ_B) is below loading line for pure shear

Modified Mohr Theory

- Safety can actually be evaluated by using a graphical approach without using the formulas .

$$n = \frac{|OQ|}{|OP|} = \frac{|OQ'|}{|\sigma'_B|} = \frac{|OQ''|}{\sigma'_A}$$



Comparison of Coulomb Mohr and Modified Mohr Theories